# U3A Maths

Term 2 / Week 2 30/04/2021

## Alternate Mean Values

Review

- Measure of Central Tendency is a "central" or "typical" value representing a given set of values.

- Arithmetic Mean ($\bar{x} = \Sigma x / N$) is also called the "Average" value in colloquial usage.

- Median is the "middle" value after arranging the set of values in Ascending order.



- Mode is the "most frequently" occuring value. This is used for discrete numeric values with fixed outcomes for ex throw of dice.
- This is also used for non-numeric values, such as, Colour, make, model of cars

- Arithmetic Mean is affected by outlying values, hence Median is invariably used as the measure of central tendency for numeric values, for ex. median house price, etc.



- Median requires a non-mathematical operation, hence it is used only as a measure of central tendency.

- Hence, Arithmetic Mean ($\bar{x}$) is extensively used for Statistical analysis.

= x =

Geometric Mean (G.M.)

$$G.M. = \sqrt[n]{x_1 . x_2 . \dots x_n}.$$

- useful when establishing an "average" or "central" value of vastly different values.

Ex: 1 Find the average of 2 m high mole hill and 5000 m high mountain.

Arithmetic Mean $= \frac{2 + 5000}{2} = 2501$ m

Note that mole hill has little effect on the average.



Geometric Mean $= \sqrt[2]{2 \times 5000} = \underline{\underline{100\text{ m}}}$.

For mole hill $= 4$ m.

G.M. $= \sqrt[2]{4 \times 5000} = 141.4$ m.

The mole hill now has a significant effect on the average!

- G.M. is also useful for Compound interest and exponential values. Used in Finance Applcns.

Ex: 2 An investment increases in value as below:

50% in the first year
20% in the second year
70% in the third year

Find the average increase / year.

1st year ⇒ 1.5
2nd year ⇒ 1.2
3rd year ⇒ 1.7

G.M. $= \sqrt[3]{1.5 \times 1.2 \times 1.7}$
$= 1.4517$

Average increase / year = 45.17%.

Airthmetic Mean $= \frac{1.5 + 1.2 + 1.7}{3} = 1.4667$

ie, 46.67% per year.

<u>Verify that the geometric Mean is the correct Value!</u>

<u>Harmonic Mean</u> $= \frac{n}{(1/x_1) + (1/x_2) + \cdots + (1/x_n)}$

- provides the correct "average" for "rate of change"!

<u>Ex:3</u> A person travels from A to B at 30 km/h and travels back at 60 km/h. What is the "average" speed?

Har. Mean $= \frac{2}{(1/30) + (1/60)} = 40$ km/h

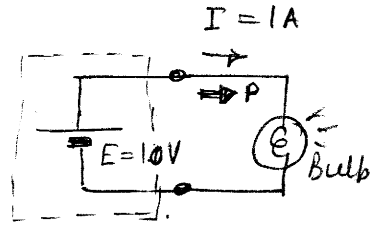
<u>Verify that H.Mean gives the correct value assuming A→B = 60 km</u>



Let us now consider an interesting electrical application.

<u>Ex.4</u>

For a D.C. Electric (Battery) system shown, calculate the power flow.




Voltage $E = 10V$;
Current $I = 1A$;

∴ Power Flow $(P) = E \times I = 10 \times 1$
$= \underline{10 \text{ Watts}}$.

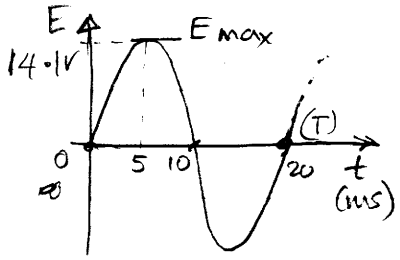
It is a 10W bulb!

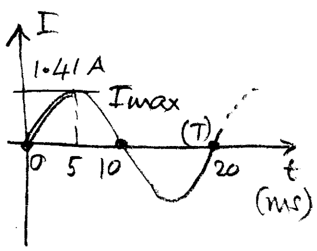
<u>Ex:5</u>

Let us consider a 50 HZ mains power source and calculate the power flow.



Both E and I vary w.r.t. time as below:




Note: for 50 Hz or cycles/second

$T = 1$ cycle time $= 1/50 = 0.02\,s$ (20 ms)

Average value over one cycle

$E_{av} = 0$, $I_{av} = 0$

∴ Power flow is zero!

- But the bulb glows even in the A.C. system.
- We use the "<u>root mean square</u>" (rms) value of E & I for power calculations.



- In fact, A.C. voltmeters and current meters (Ammeters) are designed to display R.M.S. Values!
- For sinusoidal wave, we have

$$E(t) = E_{max} \sin(2\pi f t) \quad (f = 50\text{ Hz})$$

$$E_{rms} = \sqrt{\frac{\int_0^T E^2(t)\,dt}{T}} = \left(\frac{E_{max}}{\sqrt{2}}\right) \;!!$$

Similarly; $I_{rms} = \frac{I_{max}}{\sqrt{2}}$

We have:

$$E_{rms} = \frac{14.1}{\sqrt{2}} \cong 10V$$

$$I_{rms} = \frac{1.41}{\sqrt{2}} = 1A$$

∴ Power flow $= E_{rms} \times I_{rms} = 10 \times 1 = \underline{10 \text{ watts}}$

<u>H.W.</u>: • Calculate the A.C. power flow for $E_{rms} = 230V$ & $I_{rms} = 3A$.

• Calculate $E_{max}$ & $I_{max}$.