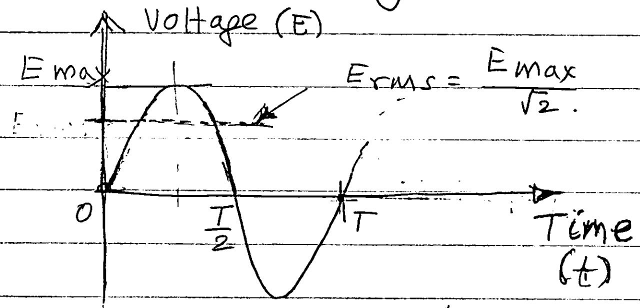
U3A - Maths

Term 2/Week 3 7/05/21

R.M.S. Value & its relevance in Elect. Engg.

- Root Mean Square (R.M.S.) Value (Root of the Mean of Squares)
  - is used extensively as a measure of voltage and current in alternating current (AC) systems.



Note: For 50 cycles/second (f = 50)

$T = 1/f = 1/50$ sec. = 20 msec

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- we have

$$E_{rms} = \frac{E_{max}}{\sqrt{2}}.$$

Let us invesigate the basis for this "magic" equation with examples in statistics and electrical engineering.

Ex. 1

Given that an A.C. voltage varies w.r.t. time as below.

$E(t) = 14.142 \sin(2\pi f t)$ volts.

where, t = time in seconds.

f - frequency = 50 Hertz

(1 Hertz = 1 cycle/sec)

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calculate voltages at time t = 0, 2.5, 5, 7.5, 10, 12.5, 15, 17.5 & 20 msec.

Soln:

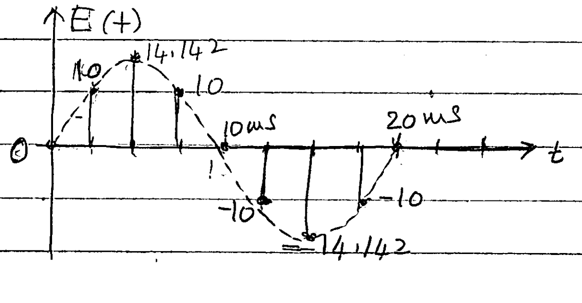
$E(0) = 14.142 \sin(0) = 0\,V$

$E(2.5ms) = 14.142 \sin(2\pi \times 50 \times 0.0025)$
$= 10\,V$

$E(5ms) = 14.142 (2\pi \times 50 \times 0.005)$
$= 14.142\,V$

similarly we have,

$E(7.5) = 10V$; $E(10) = 0V$;
$E(12.5) = -10V$; $E(15) = -14.142V$
$E(17.5) = -10V$ & $E(20) = 0V$.



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Ex. 2

Calculate the average (arithmetic mean) voltage for one cycle.

$$E_{av} = \frac{0 + 10 + 14.14 + 10 + 0 - 10 - 14.14 - 10 + 0}{9}$$

$= \underline{0}\ V$

This value is of no use for us, even though it is mathematically correct!

=X=

We define, (for N values)

$$\text{R.M.S. Value} = \sqrt{\frac{\sum_{i=1}^{N} x_i^2}{N}}$$

Note: Very soon we will be using the above equation in statistics!

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Ex. 3 Calculate the R.M.S. value for the voltage values in Ex. 2.

$$E_{rms} = \sqrt{\frac{0^2 + 10^2 + 14.142^2 + 10^2 + 0^2 + (-10)^2 + \cdots + 0^2}{9}}$$

$$= \sqrt{\frac{799.99}{9}} = \underline{9.42\ V}$$

Homework!

Mathematically exact value can be obtained using

$$E_{rms} = \sqrt{\frac{\int_0^T E_{max}^2 \sin^2(2\pi f t)}{T}}$$

This is left as "homework"

Hints: (1) $\sin^2(\theta) = \frac{1}{2}[1 - \cos(2\theta)]$

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(2) Also, integral of a sinusoid over one cycle is zero!

The final answer is

$$\left\{ E_{rms} = \frac{E_{max}}{\sqrt{2}} \right\}$$

Question

- Why the r.m.s. value is such an important (holy!) value in A.C. systems?

It is useful for electrical power calculations!

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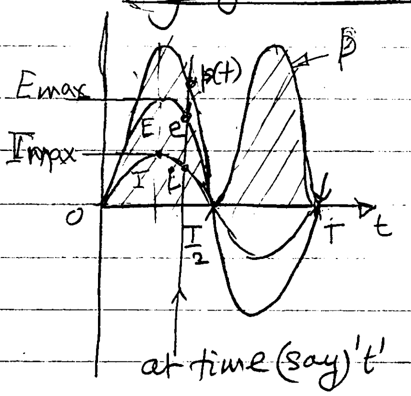
Power in A.C. Systems

We know that electrical power is defined as

$$P = E\,I \quad \text{watts}$$

This is true even for A.C. system for corresponding voltage & current at any given instant of time.



We have

$$p(t) = e(t) \cdot i(t)$$

Average power over one cycle

$$P_{av} = \frac{\int_0^T e(t)\, i(t)\, dt}{T}$$

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Expressing e(t) & i(t) sinusoidal form and integrating, we get,

$$P_{av} = \frac{E_{max} \cdot I_{max}}{2}$$

We can write above as

$$P_{av} = \left(\frac{E_{max}}{\sqrt{2}}\right)\left(\frac{I_{max}}{\sqrt{2}}\right) = \underline{E_{rms} \cdot I_{rms}}$$

- A.C. Voltmeters and current meters (Ammeters) are designed to measure and display R.M.S. values
- We no longer need to worry about the sore thumb, ie, the factor of (½) !!

Next week Pythagoras had defined Arith. Mean, Geom. Mean & Harm. Mean around 500 B.C. !!