## U3A Maths

**Term 2/Week 8** — 11-Jun-21

### Std. Dev & Z-Score

**Review**

- Population Std dev ($\sigma$)
- Sample Std. Dev. $\langle$ ($S_N$), ($S_{N-1}$)
- Bessel Correction Factor

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- Standard Deviation provides us the "measure of the spread" or "measure of dispersion"

- We have, for "Normal" distribution:
  - Mean ± 1 Std. dev ⇒ 68% of data
  - Mean ± 2 Std. dev ⇒ 95% of data
  - Mean ± 3 std. dev ⇒ 99.7% of data
  - Mean ± 4 Std. dev ⇒ 99.94% of data



- **Z-Score** provides the "distance" from the mean for a given data value.

- Z-Score is normally defined for a given population

- Z-score based analysis assumes "Normal" distribution

- Z-Score is a useful tool for Statistical analysis

Z-score for a given data value ($x$) in a population is defined as

$$Z = \frac{x - \mu}{\sigma}$$

where $\mu$ - Mean & $\sigma$ - Std. Dev.



- Note that Z-Score of
  - +1 to -1 ⇒ Mean ± 1 Std. Dev. (68% of data)
  - +2 to -2 ⇒ Mean ± 2 std. dev (95% -"-)
  - +3 to -3 ⇒ Mean ± 3 Std. dev (99.7% -"-)

**Ex. 1** Lengths of brush tail of possums have a mean ($\mu$) of 92.6 mm and a std. dev ($\sigma$) of 3.6 mm. Find the Z score for possums with tail lengths of 95.4 mm, 92.6 mm & 85.8 mm.

We have: $\mu = 92.6$ mm $\sigma = 3.6$ mm.

(a) $x_1 = 95.4$ mm

$$Z_1 = \frac{x_1 - \mu}{\sigma} = \frac{95.4 - 92.6}{3.6} = \underline{\underline{0.78}}$$

(b) $x_2 = 92.6$ mm

$$Z_2 = \frac{x_2 - \mu}{\sigma} = \frac{92.6 - 92.6}{3.6} = 0$$

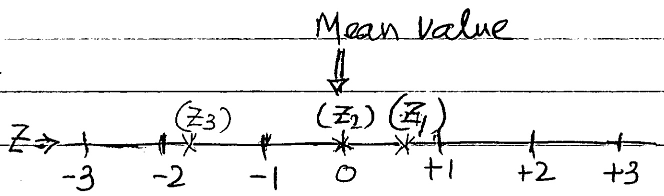


Note that Z-Score is 0 when the data value ($x$) is equal to Mean!

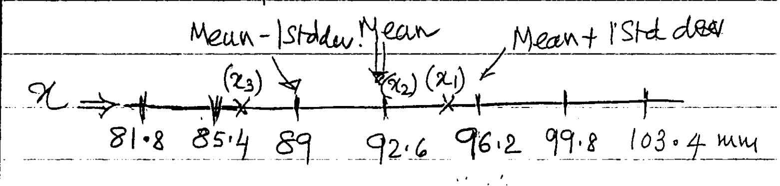
(c) $x_3 = 85.8$ mm

$$Z_3 = \frac{x_3 - \mu}{\sigma} = \frac{85.8 - 92.6}{3.6} = \underline{\underline{-1.89}}$$

Let us plot that the Z-scores on a linear scale.




Let us also plot '$x$' values.



- Hence, the Z-score helps to "standardise" the data values. Hence, it is very useful while making "inferences" (decisions) about a given data value.

**Ex.2** A school of 100 students have a mean height of 1.4 m and a std. dev. of 0.15 m. What can you infer (comment) on a student with a height of 1.86 m.

(Quite often it is difficult to "infer" with the data values, but is much



easier with Z-scores)

We have, $x = 1.85m$.

$$Z = \frac{x-\mu}{\sigma} = \frac{1.86-1.4}{0.15} = 3.07$$

We know 99.7% of students are between Z-score of $\pm 3$.

$\therefore$ the student is in the top 0.3% of the class!

- He is "probably" the tallest student in the school!!

**Ex.3** The Australian adult males have a mean height of 176 cm ($\simeq$ 5'9") with a standard deviation of 3 cms.
(must be before Indian & Chinese migration!!)



(a) Calculate the Z-score of a standard door with a height of 203 cms (6'10")

(b) Calculate the height of a person with a Z-score of +3.

(c) Check whether the person in (b) can pass through without hitting his head!

Soln

(a) $$Z_{203} = \frac{203-176}{3} = 9 \quad (!!)$$

(b) $Z = 3$ we have $Z = \dfrac{x-\mu}{\sigma}$

$$\therefore x = \mu + Z \cdot \sigma$$
$$= 176 + 3 \times 3 = 185\text{cm } (6'1")$$

(c) He can easily enter!
(Door height = 203 cm)



Home Work

Prof. Smith has marked a test. The marks (out of 60) obtained by students are:

20, 15, 26, 32, 18, 28, 35, 14, 26, 22, 17

Most students got less than 30 out of 60, hence they will fail. Prof. Smith felt that the test was hard and decided to "standardise" the marks with Z-scores. He then decided that the students with a Z-score of -1 or less would fail.

(a) How many students failed the test after standardisation?

(b) What % of students failed the test?