U3A Maths

Term 2 / Week 9 — 18-Jun-21

Grouped Frequency Table

Review $Z = \frac{x-\mu}{\sigma}$

Z-score "normalises" or "standardise" the location of the data value, making it easier for inference.

Homework

Marks ⇒ 20, 15, 26, 32, 18, 28, 35, 14, 26, 22, 17

$n = 11$

$\mu = \frac{\Sigma x}{n} = \frac{253}{11} = 23$; $\sigma = \sqrt{\frac{\Sigma (x-\mu)^2}{n}} = 6.633$

Zscore ⇒ −0.47, −1.26, 0.45, −0.75, 0.75, 1.81, −1.36, 0.45, −0.15, −0.91

(a) 2 out of 11 fail the test.

(b) % Fail = $2/11 \times 100 \cong 18\%$

Grouped Frequency Table

- When there are a large number of values in the sample it is more efficient to use "Grouped Frequency Table"

- Let us consider an Example.

Ex. 1 The age of the participants at a retirement seminar is as given below:

56, 58, 62, 62, 69, 48, 70, 71, 72, 55, 56, 58, 65, 64, 59, 60, 61, 49, 65, 70, 66, 58, 62, 67, 59, 62, 64, 63, 48, 52, 54, 74, 54, 75, 55

We have $N = 35$ (persons)

- We would like to group above data.

- How many groups?
  – the best estimate is $\sqrt{N}$.

∴ No. of groups $\cong \sqrt{35} = 5.9$ (say 6)

- We now need establish 'Group' or 'class' width
- 'Equal class width' is invariably used, except for extremities.
- We have

$x_{min} = 48$; $x_{max} = 75$

∴ class width $\cong \frac{75-48}{6}$

$= 4.5$

(say 5)

- We can write the Grouped Freq. Table as below.

	Class / Group	Frequency (f)
(*)	48 − <53	4
	53 − <58	6
	58 − <63	11
	63 − <68	7
	68 − <73	5
(*)	73 − <77	2
		$N = \Sigma f = 35$

(*) Often 'open ended' values are used for extremities, namely, <53 and >73.

- We can calculate the 'Mean' & 'Std Dev' of Grouped Freq Table:

$$\bar{x} = \frac{\Sigma f \cdot x_{mid}}{\Sigma f} \; ; \; S_N = \sqrt{\frac{\Sigma f \cdot x_{mid}^2}{\Sigma f} - (\bar{x})^2}$$

where x_{mid} is the middle value of the class.

- The revised Group Freq Table is as below:

x_{mid}	f	$f \cdot x_{mid}$	$f \cdot x_{mid}^2$
50.5	4	202.0	10,201.00
55.5	6	333.0	18,481.50
60.5	11	665.5	40,262.75
65.5	7	458.5	30,031.75
70.5	5	352.5	24,851.25
75.5	2	151.0	11,400.50
	35	2,162.5	135,228.75

$\therefore$ we have:

$\Sigma f = 35$; $\Sigma f \cdot x_{mid} = 2162.5$;
$\Sigma f \cdot x_{mid}^2 = 135,228.75$

$$\text{Mean } (\bar{x}) = \frac{\Sigma f x_{mid}}{\Sigma f} = \frac{2162.5}{35} = 61.79$$

$$\text{Std. Dev } (S_N) = \sqrt{\frac{\Sigma f \cdot x_{mid}^2}{\Sigma f} - (\bar{x})^2}$$

$$= \sqrt{\frac{135228.75}{35} - (61.79)^2}$$

$$= \sqrt{45.68} = 6.759$$

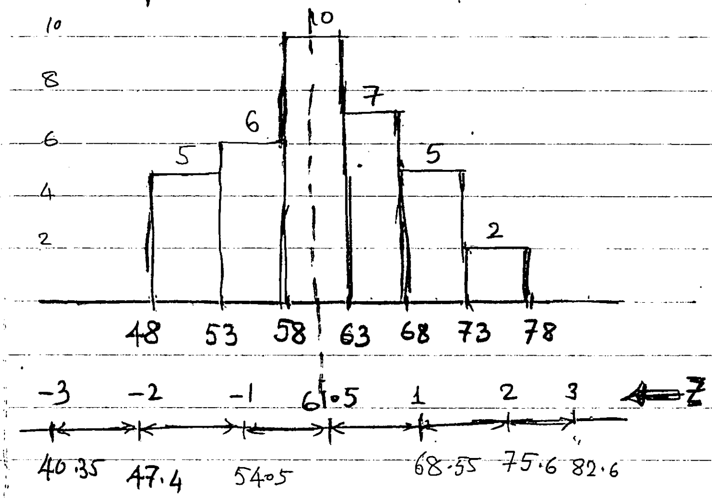
Notes

- The values of Mean & Std. Dev calculated from "raw data" are as given below:

$\bar{x} = 61.23$ & $S_N = 7.067$

- The results from Grouped Freq Table are quite close. It will be even better for a larger sample and smaller class width.

Let us now plot the Grouped Freq. Table as a bar chart!



- All values are within 3 Std. Dev or $Z = \pm 3$

- The above plot is called a "Histogram".
- Note that "Histogram" is NOT a "Normal Distribution Curve"!!

Homework (in cm)

The lengths of leaves collected from an oak tree are as given below:

9, 16, 13, 7, 8, 4, 18, 9, 12, 5, 9, 9, 16, 1, 8, 17, 1, 10, 5, 9, 11, 15, 6, 14, 9, 1, 12, 5, 16, 4, 16, 8, 15, 14, 17, 18, 10, 17. (38 values)

(a) Prepare grouped frequency table
(b) Calculate Mean ($\bar{x}$) and Std. Dev (S_N)
(c) Draw the Histogram.

Ian King's suggested video

- https://www.youtube-nocookie.com/embed/Zh3Yz3PiXZw

- You can also search in 'youtube' for:

'Alternative Math Short Film'