U3A Maths

18-Feb-2022 Term 1 / Week 3

Z - Score

Review of last week

- Analysis of "sample means" from a given population, namely, $\{1, 2, 3, \ldots, 99, 100\}$.
  - Considering Trial Run #1 Results.

Sample Size (N) = 10

Sample →	#1	#2	#3	#4	#5	#6	#7
Sample Mean ($\bar{x}$) →	66.6	37.5	48.1	37.2	52.2	45.8	50.3

Mean of Sample Means $(\bar{\bar{x}}) = 48.24$

- Calculate Std. Dev. of Sample Means

$(\bar{x} - \bar{\bar{x}}) \Rightarrow 18.36, -10.74, -0.14, -10.74, 3.96, -2.44, 2.06$

Std Dev of Sample Means $(\sigma_{\bar{x}}) \Rightarrow \sqrt{\dfrac{\sum(\bar{x}-\bar{\bar{x}})^2}{N_{\bar{x}}}} = \sqrt{\dfrac{593.68}{7}} = 9.21$

-2-

- The population Mean $(\mu) = 50.5$

Population Std Dev. $(\sigma) = 28.87$

Note that the population data values are NOT 'normally' distributed

- However the sample means $(\bar{x})$ values are "normally" distributed with Mean $(\bar{\bar{x}}) = \mu$ (same as pop. Mean)

and Std dev $(\sigma_{\bar{x}}) \cong \dfrac{\sigma}{\sqrt{N}}$

(Central Limit Theorem!)

- Let us check Test Run #1 Results

$\sigma_{\bar{x}} = 9.21$

$\sigma/\sqrt{N} = \dfrac{28.87}{\sqrt{10}} = 9.13$

Considering the small sample size, the results are comparable!

-3-

- Let us also check "normality" of sample means.

$\bar{\bar{x}} \pm 1$ Std Dev $= 48.24 \pm 9.21$
$= 39.03$ to 57.45

$\bar{\bar{x}} \pm 2$ Std Dev $= 48.24 \pm 2 \times 9.21$
$= 29.42$ to 66.66

- Note that 100% of all sample means in Test Run #1 fall in the range of 29.42 to 66.66

- As per normal distribution, 95% of the data values are within Mean ± 2 × Std Dev.

- The distribution of Sample Means is close to "Normal Distribution"

-4-

Z - Score

- Z-Score provides the distance from the mean for a given data value in a standardised way

- Z-Score is useful when analysing the data values — especially for normally distributed population.

- Z-Score is a useful tool for statistical analysis

- Z-score is normally defined for a population hence, mean (μ) & Std Dev (σ) are used.

- Z-score for a given data value (x) in a population is defined as below:

$$Z = \frac{x - \mu}{\sigma}$$

where x - data value
μ - Mean of population
σ - Std Dev of population

- For normal distribution, a Z-score of
  - +1 to −1 ⇒ Mean ± 1 Std Dev {68% of data}
  - +2 to −2 ⇒ Mean ± 2 Std Dev {95% of data}
  - +3 to −3 ⇒ Mean ± 3 Std Dev {99.7% of data}
  - +4 to −4 ⇒ Mean ± 4 Std Dev {99.99% of data}

Ex.1 Lengths of brushtail of possums has a mean (μ) of 92.6 mm and Std. dev (σ) of 3.6 mm.
Find Z-scores for possums with tail lengths of 95.4 mm, 92.6 mm and 85.8 mm

Solution

we have $\mu = 92.6$ mm
$\sigma = 3.6$ mm

(a) $x_1 = 95.4$ mm,

$$Z_1 = \frac{x_1 - \mu}{\sigma} = \frac{95.4 - 92.6}{3.6} = 0.78$$

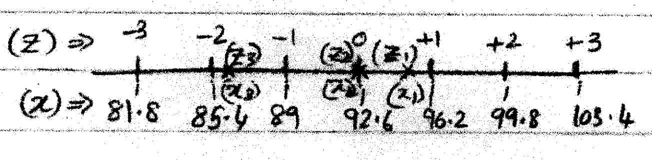
(b) $x_2 = 92.6$ mm

$$Z_2 = \frac{x_2 - \mu}{\sigma} = \frac{92.6 - 92.6}{3.6} = 0$$

(c) $x_3 = 85.8$ mm

$$Z_3 = \frac{x_3 - \mu}{\sigma} = \frac{85.8 - 92.6}{3.6} = -1.89$$

Let us plot the Z-scores on a linear scale.



—x—

Ex.2 The Australian adult males have a mean height of 176 cms (≈ 5'9") with a std. dev. of 3 cms (must be before chinese & Indian migration)

(a) Calculate the Z-score of a Standard door with a height of ~~203~~ 204 cms (~~6'[redacted]"~~) (6'8")

(b) Calculate the height of a person with a Z-score of +3

(c) Check whether the person in (b) can pass through without hitting his/her head!

Solution

(a) $$Z_{204} = \frac{204 - 176}{3} = 9.333 \; (!!)$$

[redacted]

(b) For $Z = 3$ find 'x' using

$$Z = \frac{x - \mu}{\sigma}$$

$$\therefore x = \mu + Z\sigma = 176 + 3 \times 3$$
$$= \underline{\underline{185 \text{ cms}}} \; (6'1")$$

(c) The person can easily enter even with a hat!!