U3A Maths (No class on 27th May)

20-May-2022 Term 2 / Week 4

Normal Distribution (Cont'd)

Home work!

Why do we need $\frac{1}{\sqrt{2\pi}}$ & $\frac{1}{2}$?

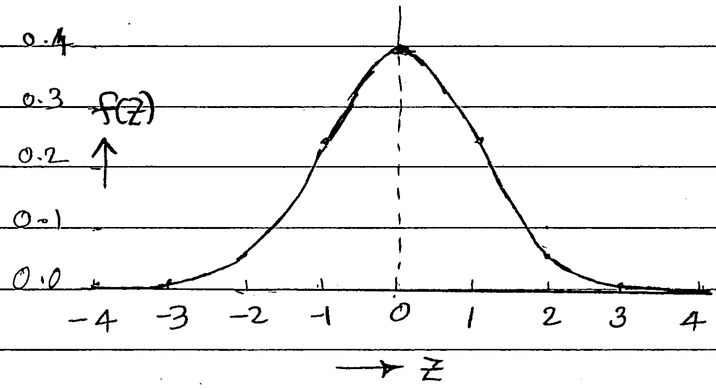
- As an example to illustrate bell shaped curve, we used

$$f(x) = e^{-x^2}$$

- However, the function which correctly represents "Standard Normal Distribution" is as below.

$$f(z) = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{1}{2} \cdot z^2}$$

- Note that usage of Z-score "Standardises" the Normal distribution curve, and it also results in a simpler equation.
- Of course, the z value is obtained using the population mean (μ) and Std. dev (σ).
- Let us now plot the function $f(z)$ for z values varying from -4 to $+4$.



z		$f(z) = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{1}{2}z^2}$
	0	0.3989
(-1)	1	0.2419
(-2)	2	0.0540
(-3)	3	0.0025
(-4)	4	1.888×10^{-7} ($\simeq 0$)

- Note that we are NOT interested in the function value itself, but the area under the curve.
- We know that the area under the curve corresponds to "relative frequency", in other words the "probability"
- We need to integrate the function to get the area!
- Let us integrate from $-\infty$ to ∞ to obtain the total area under the curve.

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- We have

$$\text{Total Area} = \frac{1}{\sqrt{2\pi}}\left[\int_{-\infty}^{+\infty} e^{-\frac{1}{2}z^2}\, dz\right]$$

- There is no simple way to establish the above integral! However the final result is very simple!!

- $$\text{Total Area} = \frac{1}{\sqrt{2\pi}}\left[\sqrt{2\pi}\right] = 1$$

- We now have the total area under the curve, which is 1! (100% probability!)

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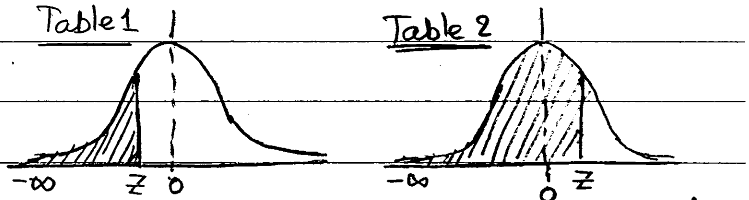
- However, we need the area under the curve for any two given values of 'z' (say $z = -1$ to 1, etc)

- In practice, the area under the curve is given as a table in Appendix of Statistics books!

- The areas are typically provided in 2 tables for given z-values



(Negative z values) (-3.5 to 0) (Positive z values) (0 to +3.5)

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Ex.1 Calculate the following probability values (area under the curve) for given z values

(a) $P[z \leq -1.25]$ (b) $P[z > 1.25]$

(c) $P[z: -0.38 \text{ to } +1.25]$

(d) $P[z: -1 \text{ to } +1]$

Ans.

(a) $P[z \leq -1.25] = 0.1056$

(b) $P[z > 1.25] = 1 - P[z \leq 1.25]$

$= 1 - 0.8944 = 0.1056$

(c) $P[z: -0.38 \text{ to } 1.25]$

$= P[z \leq 1.25] - P[z \leq -0.38]$

$= 0.8944 - 0.3520$

$= 0.5424$

(d) $P[z: -1 \text{ to } +1]$

$= P[z \leq +1] - P[z \leq -1]$

$= 0.8413 - 0.1587 = 0.6826$

Note: this is $\pm$ 1 std dev $\Rightarrow$ 68.26%

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Ex.2 The breakdown voltage (X) of a randomly chosen diode of a particular type is known to be normally distributed with mean $(\mu) = 40$ V and Std. dev $(\sigma) = 1.5$ V.

What is the probability of the chosen diode breakdown voltage is between (a) 39V to 42V. (b) $\leq$ 39V.

Soln (a) $X_1 = 39V \quad \therefore z_1 = \frac{39-40}{1.5} = -0.6667$

$X_2 = 42V \quad \therefore z_2 = \frac{42-40}{1.5} = 1.3333$

$\therefore P[x1 : x2] \Rightarrow P[z1 : z2]$

$= P[-0.6667 : +1.3333]$

$= P[\leq 1.3333] - P[\leq -0.6667]$

$= 0.9082 - 0.2514 = 0.6568$

(b) $P[z \leq -0.6667] = 0.2514$