U3A-Maths

1-Jul-2022 Term 2 / Week 10

Central Limit Theorem - Examples

Ex.1 Mobile phone bills for residents in a city have a mean (μ) of \$65 and a Std. dev. (σ) of \$10. The data is known to have "Normal Distribution". Calculate the probability that phone bill of an individual is greater than \$70.

We have $\mu = 65$, $\sigma = 10$ & $x = 70$

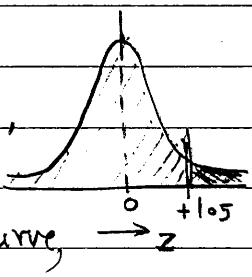
To Find $P[x > 70]$

-2-

Calculate $z = \dfrac{x-\mu}{\sigma} = \dfrac{70-65}{10}$

$= \dfrac{5}{10} = 0.5$

Since the data is Normally distributed, we can use "Standard Normal Probability" curve, using the z value.



From Table

$P[z > 0.5] = 1 - P[z \leq 0.5]$

$= 1 - 0.6915$

$= 0.3085$ (30.85%)

$\therefore P[\text{Bill} \geq \$70] = \underline{30.85\%}$

-3-

- Let us say that the data distribution in Ex-1 is unknown or known to be NOT noramally distributed.
- However, the population Mean (μ) and Std. Dev (σ) can still be evaluated from the population data.
- But we cannot use "Standard Normal Table" for calculating the probability for a given data value (x)
- This is when the Central Limit Theorem becomes useful.

-4-

- Central Limit theorem enables calculation of the probability of the Mean value of a given sample ($\bar{x}$) instead of probability of a given data value (x).
- As per CLT, in general, the sample size (n) should be ≥ 30. However, it can be less depending on the "Normalness" of the data distribution.
- Lower the sample size when the distribution is closer to "Normal Distribution"
- For perfect Normal Distribution Sample size (N) can be 1 as in Ex-1 !!

Ex.2 The population in Ex.1 is given to be NOT normally distributed.
Calculate the probability of obtaining a Mean Value ($\bar{x}$) of \$70 for a given sample of 36 bills.

We have: $\mu = 65$; $\sigma = 10$
$n = 36$; $\bar{x} = 70$

From Central Limit theorem, sample means are normally distributed with:
Mean of Sample Means ($\mu_{\bar{x}}$) = 65
and Std. dev. of Sample Means ($\sigma_{\bar{x}}$) = $\sigma/\sqrt{n}$

$\therefore \sigma_{\bar{x}} = \frac{10}{\sqrt{36}} = \frac{10}{6} \simeq 1.6667$

We need to find $P[\bar{x} > 70]$

we have

$$Z_{\bar{x}} = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{70-65}{1.6667}$$

$$= 2.9999 \ (\simeq 3.0)$$

$$P[Z_{\bar{x}} > 3.0] = 1 - P[Z_{\bar{x}} \leq 3.0]$$
$$= 1 - 0.9987$$
$$= 0.0013 \ (0.13\%)$$

$\therefore P$[Mean Bill for Sample $\geq$ \$70] = 0.13%

Note: The probability is much lower, since Sample Mean "averages out" individual bills!

Ex.3 (From Week 8)
The nicotine content in a cigarette of a particular brand is found to have a population mean (μ) of 0.8 mg and a popln std dev (σ) of 0.1 mg. If an invidual smokes 100 cigarettes in a week, what is the probability the total amount of nicotine consumed is $\geq$ 82mg/week.

We have: (for population)
$\mu = 0.8$ mg; $\sigma = 0.1$ mg
Sample size (n) = 100
Sample Mean ($\bar{x}$) = $\frac{82}{100} = 0.82$ mg
(ie, 82 mg consumption for 100 cigarettes)

From Central Limit theorem:

$\mu_{\bar{x}} = 0.8$ mg $\quad \sigma_{\bar{x}} = \frac{0.1}{\sqrt{100}} = 0.01$ mg

To Find: $P[\bar{x} \geq 0.82 \text{ mg}]$

We have $Z_{\bar{x}} = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{0.82 - 0.80}{0.01}$

$$= +2.0$$

$$P[Z_{\bar{x}} > 2] = 1 - P[Z_{\bar{x}} \leq 2]$$
$$= 1 - 0.9772$$
$$= 0.0228 \ (2.28\%)$$

$\therefore$ Probability of Consuming $\geq$ 82 mg/week = 2.28%