U3A Maths

3-Jun-2022 Term 2 / Week 6

Normal Distribution Examples

- We now know how to use std. Normal distribution table — which provides the cumulative probability (area under the Gaussian curve) for a given Z-score or Z-score range.

- We will now do some practical examples — which illustrate how to calculate the probability.

Ex-1 Cholestrol Levels

Total cholestrol levels for men (in US) in 35-44 age group has a mean (μ) of 2.360 mmol/l and a std. dev. (σ) of 0.427 mmol/l.

(a) what percentage of men have a total cholestrol level less than 2.540

(b) If 250 men are randomly selected, how many would you expect to have cholestrol level greater than 2.935

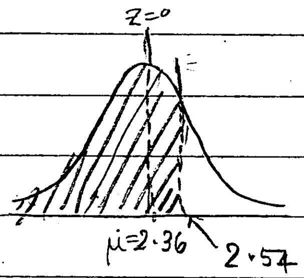
(Note: In US mg/dl is used
~~In AUS mmol/l is used~~
mmol/l = mg/dl $\times$ 0.01129)

(a)

$\mu = 2.360$

$\sigma = 0.427$

$P[<2.54]$



(Note: the rough sketch helps usage of appropriate table)

We have: $x = 2.54$ $\therefore Z = \dfrac{x-\mu}{\sigma} = \dfrac{2.54-2.36}{0.427}$

$= 0.422$

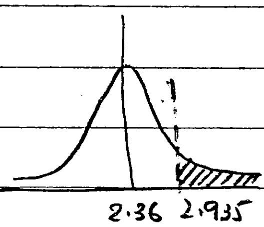
From table

$P[Z \le 0.422] = \underline{0.6635}$ after interpolation bet 0.42 & 0.43

$\therefore P[<2.54] = \underline{\underline{66.35\%}}$

(b)

$P[>2.935]$ as shown in sketch



$x = 2.935$;

$Z = \dfrac{x-\mu}{\sigma} = \dfrac{2.935 - 2.36}{0.427} = 1.347$

$P[Z>1.347] = 1 - P[Z \le 1.347]$ ← From Table

$= 1 - 0.9110 = \underline{0.089}$

(8.9%)

$\therefore$ No. of men with total cholestrol level > 2.935

$= 0.089 \times 250$

$\simeq \underline{\underline{22}}$

Ex.2 Tyre Warranty

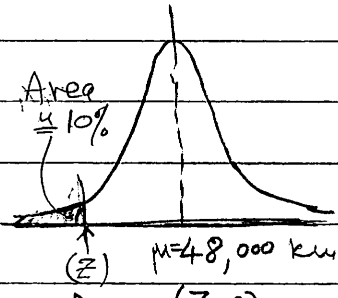
A brand of car tyre has a life expectancy that is normally distributed, with a mean (μ) of 48,000 km and a Std. Dev (σ) of 4,000 km. How should you word the warranty, if the free replacement is to be kept under approximately 10% of the total sales.

We have:

$\mu = 48{,}000$ km

$\sigma = 4000$ km.

$\therefore$ Probability ≤ 0.1



- From the table find the Z-value for the area corresponding to 0.1

$\therefore Z = -1.28$ $\boxed{\text{Area} \cong 0.1003}$

We have

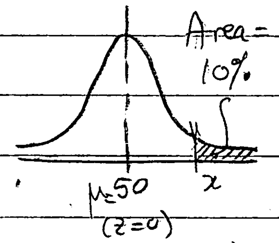
$$x = \mu + Z\sigma$$
$$= 48{,}000 + (-1.28) \times 4000$$
$$= \underline{42{,}880} \text{ km.}$$

Warranty for approx 40,000 or 45,000 km ??

Ex.3

Scores for California Peace officer test are normally distributed with $\mu = 50$ & $\sigma = 10$. An agency will only hire applicants with scores in the top 10%. What is the lowest score which is eligible for hire..

$\mu = 50$; $\sigma = 10$



From the table; we need to find the Z-value for cum. area $> (1-0.1) = 0.9$

$\therefore Z \cong 1.28$

$\therefore x = \mu + Z\sigma = 50 + 1.28 \times 10$

$= \underline{62.8}$

Say the lowest score is $\underline{63}$.

Homework

In a large statistics class, the final exam scores are normally distributed with a mean of 72 and a standard Deviation of 9. Grades are to be assigned as follows:

- Grade A – Top 10%
- Grade B – Next 20%
- Grade C – Middle 40%
- Grade D – Next 20%
- Grade F – Bottom 10%

Find the lowest score that qualify a student for A, B, C & D.