U3A - Maths

10 - Jun - 2022 Term 2 / Week 7

Central Limit Theorem

- We now know how to calculate the probability using "Standard Normal Probability distribution" table.
- We could also solve some practical problems, such as Tyre warranty!
- Of course, we assumed "Normal" or "Gaussian" distribution.
- This may not be true in practice!

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- Central Limit Theorem can be used for all types of distribution

Central Limit Theorem

1. If samples of size 'n' $(n \geq 30)$ are drawn from any population with a mean (μ) and std dev (σ), then sample means have approximates to Normal Distbn! The greater the sample size, better is the approximation.
2. The above is valid even if the population is NOT Normally distributed.!!

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3. In addition,
   - Mean of Sample Means $(\mu_{\bar{x}}) = \mu$
   - Std. Dev of Sample Means $(\sigma_{\bar{x}})$ $= \dfrac{\sigma}{\sqrt{n}}$

- Central Limit theorem provides a powerful tool to carry out inferences in practice.

- For the example data set given
  - Verify $\mu_{\bar{x}}$ and $\sigma_{\bar{x}}$
  - Prepare a Frequency Table to verify the distribution of Sample Means.

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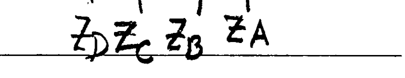
Homework solution (Week 6)

$\mu = 72$ marks

$\sigma = 9$ marks



We have:

Grade A : $\geq Z_A = 1.28$; $X_A \doteq 83.5$

Grade B : $\geq Z_B = 0.525$; $X_B = 76.7$

Grade C : $\geq Z_c = -0.525$; $X_c = 67.3$

Grade D : $\geq Z_D = -1.28$; $X_D = 60.5$

Grade F : $< Z_D \Rightarrow X_D < 60.5$

A student getting 60.5% mark FAILS!

Central Limit Theorem for Uniform Distribution (6 sided Dice Example)

Samples - 25 samples each with 25 values

3	6	6	3	2	5	1	6	6	5	3	5	2	4	1	3	6	3	4	6	1	3	6	5	1
4	6	1	2	5	3	6	5	6	1	5	2	5	3	5	4	1	1	5	1	3	4	6	1	5
2	3	5	3	6	5	5	1	6	3	3	1	2	1	5	3	4	4	5	1	6	6	6	1	2
5	1	2	3	3	3	1	1	3	1	1	1	1	6	2	4	2	5	3	2	4	3	5	6	2
4	5	2	1	6	4	4	6	2	2	3	1	4	5	5	4	6	2	3	1	3	2	6	1	5
6	4	1	2	2	5	6	3	6	1	4	1	4	6	5	4	6	3	6	6	2	5	3	1	3
6	4	6	2	4	1	5	3	4	1	4	6	1	4	1	5	4	6	3	1	1	5	1	3	4
1	5	5	3	3	6	1	1	4	4	4	6	5	3	1	5	3	2	2	3	3	6	3	6	6
6	5	2	1	6	5	6	5	5	1	4	4	5	1	5	4	1	2	2	1	4	2	1	1	2
2	3	5	5	2	5	3	3	2	4	1	4	4	3	5	4	2	2	4	5	3	1	1	2	5
1	6	1	4	6	3	6	3	1	6	3	5	6	1	2	2	4	3	5	3	4	3	5	2	2
1	5	6	5	1	3	2	6	3	6	5	3	6	6	5	5	3	6	2	5	6	4	4	5	5
4	5	3	6	3	2	1	6	4	3	1	5	1	4	1	5	3	5	5	6	1	6	6	4	4
2	2	1	4	3	5	3	2	2	5	5	4	3	4	5	3	4	2	2	6	4	6	3	4	2
4	5	4	3	3	4	5	5	4	6	6	6	5	2	4	1	5	4	3	3	5	5	6	4	2
4	6	2	3	3	1	4	2	2	4	6	1	1	1	1	2	6	2	3	1	6	6	3	3	2
1	5	3	4	2	4	4	1	1	3	6	2	1	4	1	4	3	2	2	2	1	6	6	1	4
5	5	2	3	4	1	2	1	6	4	2	4	4	4	5	3	3	3	4	6	6	3	1	4	6
6	5	4	5	1	6	3	4	6	1	6	4	6	5	4	1	4	5	4	4	2	3	5	6	3
3	5	1	2	6	6	4	5	3	5	1	3	1	6	4	5	2	4	5	5	4	4	4	2	6
2	1	3	1	2	6	4	3	2	6	6	4	3	1	2	1	4	2	6	5	2	5	2	5	3
2	1	4	4	1	5	2	4	2	2	4	6	2	1	6	2	6	5	4	6	6	3	1	5	4
5	2	6	1	5	6	1	1	1	1	3	3	4	4	2	2	2	2	2	5	3	5	2	4	6
5	2	5	1	3	5	4	5	2	6	6	3	2	1	3	5	5	1	4	6	6	6	1	1	1
1	4	2	1	5	3	3	2	3	3	5	4	4	1	5	2	4	5	1	4	4	5	1	5	1
3.40	4.04	3.28	2.88	3.48	4.08	3.44	3.36	3.44	3.36	3.88	3.52	3.28	3.24	3.40	3.32	3.72	3.24	3.56	3.76	3.60	4.28	3.52	3.28	3.44

Sample Means for current data set!

Sample Means

3.32	2.92	4.12	3.92	3.32	3.48	4.20	3.64	4.08	3.28	2.88	3.80	3.44	3.68	3.80	3.24	3.76	3.56	3.56	3.40	3.96	3.36	4.16	3.24	3.36

Note: This row is from the previous set of random numbers, since Excel continuously changes the random data set!

Mean of Sample Means $\boxed{3.58}$ **Std Dev of Sample Means** $\boxed{0.37}$

Notes:

1. When 6 sided dice is thrown a number of times:
   Population Mean = 3.55 [Mean of Sample Means = Population Mean as per Central Limit Theorem]
   Population Std Dev = 1.707 [Std Dev of Sample Means = Pop Std Dev / sqrt(sample size) as per Central Limit Theorem]

2. Note that if we prepare a Frequency Table for the Population, we will obtain an "Uniform Distribution"!

3. Preapare a Frequency Table for Sample Means and check the Distribution.