U3A - Maths

17-Jun-2022 Term 2 / Week 8

Central Limit Theorem - Cont'd

- Central Limit Theorem provides a method to analyse a population which may or may not be normally distributed!

- The "trick" is to deal with "Mean Values" of samples.

- Central Limit Theorem assumes that population Mean (μ) and population Std. Dev. (σ) are known.

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- The Central Limit theorem states that the "Sample Means" ($\bar{x}$) are Normally distributed with a Mean ($\mu_{\bar{x}}$) = μ and a Std. dev ($\sigma_{\bar{x}}$) = $\sigma/\sqrt{n}$

  where 'n' is the sample size

- Sample Size requirement depends on the "Normality" of the population. However, a value of $n \geq 30$ gives good results even for non-normal population!

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- Last week, we considered an example of "dice throw" to illustrate the validity of central limit theorem - using 25 samples each with a sample size of 25 randomly generated value from 1 to 6 using Excel spreadsheet.

- We prepared a Freq. Table to check the 'normal distribution' of sample means

- We were reasonably impressed by the results!

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- Note that we considered 25 samples only to illustrate the validity of Central Lim. Th.

- Finally, if we accept the validity of Central Limit theorem,
  - we can analyse and make inferences about the sample, considering a single sample!

- We can analyse the "Sample mean", just the way we analysed the values in the Normally distributed population!

Ex. 1

The nicotine content in a cigarette of a particular brand is found to have a population mean (μ) of 0.8 mg and a population std dev (σ) of 0.1 mg. If an individual smokes 100 cigarettes in week, what is the probability that the total amount of nicotine consumed per week is at least 82 mg.

We have:

$\mu = 0.8$; $\sigma = 0.1$

Sample size $(n) = 100$

For 82 mg total consumption

∴ Mean of the sample $(\bar{x}) = \frac{82}{100}$

$= 0.82$ mg.

As per Central Limit theorem, sample means are normally distributed with

$$\mu_{\bar{x}} = \mu = 0.8 \text{ mg}$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.1}{\sqrt{100}} = 0.01 \text{ mg}$$

We have, for our sample;

∴ Sample Mean $(\bar{x}) = 0.82$ mg

$$\therefore Z_{\bar{x}} = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{0.82 - 0.80}{0.01}$$

$$= +2$$

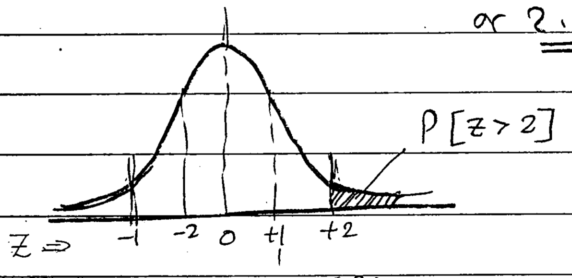
∴ $\text{Prob}[\bar{x} \geq 0.82] \Rightarrow \text{Prob}[Z_{\bar{x}} \geq +2]$

From Standard Normal Table

$\text{Prob}[Z_{\bar{x}} \geq +2] = 1 - 0.9772$

$= 0.0228$

or 2.28%



∴ Probability of consuming at least 82mg is 2.28%

- Who enunciated the Central Limit theorem?
  - Abraham de Moivre (in 1733)
    (26-May-1667 - 27-Nov-1754)
  - Moved to France to England after persecution of Huguenots (Protestants) in 1685.
  - Friend of Newton & Halley.
  - De Moivre's Formula!!
    $(\cos x + i \sin x)^n = \cos(nx) + i \sin(nx)$
    which was later refined by Euler $[e^{ix} = \cos x + i \sin x]$
- CLT was later refined by Laplace (French) (1812) & a proof was provided by Lyapunov (Russian - 1901)