U3A Maths

3-Feb-2023 Term 1 / Week 1

Complex Numbers

- A Complex Number is written as below:

$$(a+bi) \text{ or } (a+ib)$$

where:
a – Real Part (Real Number)
bi or ib – Imaginary Part
where, $i = \sqrt{-1}$ is called the Imaginary Number!

- An "Imaginary Part" is essentially "square root of a negative number"

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For Example,

- $\sqrt{-9} = \sqrt{(-1)(9)} = \sqrt{(-1)} \cdot \sqrt{(9)}$
$= i3$ or $3i$

- $\sqrt{-54} = \sqrt{(-1)(54)} = \sqrt{(-1)} \cdot \sqrt{(54)}$
$= i7.348$ or $7.348i$

Hence 'bi' or 'ib' represents 'square root of a negative number'.

- The term "imaginary number" is not only confusing, but it is also misleading!

- The fact is, we encounter "square root of a negative number" while solving mathematical equations!

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- Let us first explore the so called "Real" numbers and explore how real they are!

- At the conclusion of it, we will show that "Complex Numbers" are "the most general form of numbers" and also the "most versatile"

- In fact the "True nature of Reality (Universe)" can only be represented by Complex Numbers – as per Quantum Mechanics Eqn!!!

$$\hat{H}\psi(r,t) = i\hbar\frac{\partial}{\partial t}\psi(r,t)$$
(where $i = \sqrt{-1}$)

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- Let us now explore with the evolution of numbers.

- Let us start with the set of numbers as below

$$\mathbb{N} = \{1, 2, 3, \ldots\ldots\}$$

The above set is called "Set of Natural Numbers" and denoted by symbol '$\mathbb{N}$'.

- Let us do "arithmetic operations" on the above numbers
  – Addition of numbers in the set generates a number in the set
  Ex: $2+3=5$, $5+10=15$ etc.

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- Hence, mathematically, we say that the set of natural numbers is "closed" for addition operation
- Obviously this set is "closed" for multiplication. etc.
  $2\times3=6$; $4\times5=20$; etc.
- But it is NOT "closed" for subtraction!
  $4-2=2$ ✓ ; $5-5=0$ ×
  $4-8=-4$ ×
- We now need to extend the set (N) to include zero and negative numbers.

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- The new set is called "Integers"
  $\{I\} = \{\ldots -3,-2,-1,0,1,2,3,\ldots\}$
- It is a huge step in mathematics to accept the introduction of such new numbers, especially if they have no practical relevance or use.
- Ironically, zero and negative numbers were prevalent in early civilizations, such as, India, China, Babylonia etc
- The greek philosophers were aware of them but they were not comfortable with them. Zero is Nothing! Negative is False!!

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- Romans being very practical completely eliminated zero and negative numbers from their number system! For them it was impractical & fictious number — in other words, they were "imaginary"(!) numbers.
- Thanks to the Romans, Western world had to wait till 1200 AD for the introduction of decimal system by Fibonacci. He introduced "modus Indorum" (method of Indians) in a manuscript called Liber Abaci (1202 AD)

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- Negative numbers had to wait much longer! Even Leibniz (1646-1716 AD) held that negative numbers were invalid but still used them in his calculations. The final result always had to be "positive"! Negative numbers were first documented by Cardano (Italy) in his book "Ars Magna" in 1545 AD.
- Incidentally, the earliest detailed document on mathematics which includes, decimal system, negative numbers, arithmetic equations for use of zero, solution to quadratic equations etc, was written 628 AD by Brahma Gupta in India. It was called "Doctrine of Brahma".
  Brahma ⇒ Universal Energy or God!

(To be cont'd)