V3A Maths

10-Feb-2023 Term 1/Week 2

## Intro to Complex Numbers (2)

### Review

- Complex number $(a+ib)$ or $(a+bi)$
  where $i = \sqrt{-1}$ - imaginary No.!!
- 'bi' or 'ib' is essentially "square root of a negative number"
  For Ex: $\sqrt{-9} = \sqrt{-1}.\sqrt{9} = \sqrt{-1}\times 3 = i3$ or $3i$
- Complex Number arithmetic can be done using usual algebraic methods:

Ex: $(2+3a)+(5+2a) = 7+5b$



Similarly

$(2+3i)+(5+2i) = (7+5i)$

- Evolution of numbers:

Set of Natural Numbers

$\{\mathbb{N}\} = \{1, 2, 3, 4, \ldots\}$.

- 'N' is closed for addition not for subtraction.

Ex: $2+3=5 \Rightarrow$ Result is in $\{\mathbb{N}\}$

$2-2=0 \Rightarrow$ Result not in $\{\mathbb{N}\}$

$4-6=-2 \Rightarrow$ Result not in $\{\mathbb{N}\}$

- We now need to include zero & Negative numbers in the set to "close" the set



- The new set is called "Set of Integers"

$\{\mathbb{Z}\} = \{\ldots -3, -2, -1, 0, 1, 2, 3, \ldots\}$

- Thanks to Romans, western world had to wait till
  - 1200 AD - decimal system & zero
  - 1545 AD - negative numbers

were used in Maths calculations. However, only in late 1700s they became more acceptable! (in fact $\sqrt{-1}$ was used by Cardano in 1545 - Ars Magna)

- Earliest detailed document on mathematics including decimal numbers, negative numbers, quadratic equation was written in 628 AD by BrahmaGupta in India!



- Set of integers ($\mathbb{Z}$) is closed for addition, multiplication & subtration but not for division

for Ex $2/5$, $1/2$, $-2/3$ etc

The above numbers are called "Rational Numbers" (Ratio of numbers)

- We can now define a new set $\{\mathbb{Q}\}$ to include rational numbers

$\{\mathbb{Q}\} = \{\ldots -3, -2, -1, 0, 1, 2, 3, \ldots$
$\ldots 1/2, 1/3 \ldots -1/2, -2/3, \ldots\}$

The above set is closed for addition, Subtraction, Mult. & Division

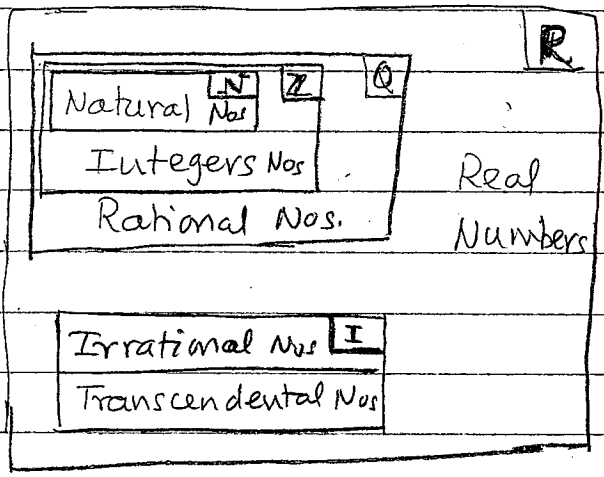
- In addition, we still need to numbers, such as, $\sqrt{2}$, $\sqrt{3}/2$, $-\sqrt{3}/2$, etc. Such a set is called set of Irrational Numbers $\{\mathbb{I}\}$

$\{\mathbb{I}\} = \{\ldots \sqrt{2}, \sqrt{5}/2, -\sqrt{3}/2 \ldots\}$

- In addition we also need to include "Transcendental numbers", namely $e$, $\pi$, $2^{\sqrt{2}}$ etc as a part of irrational number.
- We now finally have a "Set of Real Numbers" $\{\mathbb{R}\}$



- We now define a set of Real numbers ($\mathbb{R}$) as below:




- The set of real numbers is closed for all arithmetic operations !?



- However, the set of real numbers does not include "square root of a negative number", For Ex $\sqrt{-9}$, $\sqrt{-1}$ etc
- Hence, to complete the number system we need to include $\sqrt{-1}$ or alternatively a Complex number. $(a+ib)$ where $i=\sqrt{-1}$
- A "set of complex numbers" is denoted by $\{\mathbb{C}\}$



- We can show that a set of complex numbers is closed for all arithmetic operations
- In other words, any arithmetic operation involving complex numbers will always result in a complex number !!
- Hence, the most general form of a number is a "Complex Number" !!
- "Imaginary Number" ($i$ or $\sqrt{-1}$) is the missing piece to complete the number system!