U3A Maths

24-Feb-2023 Term 1/Week 4

Imaginary Numbers - Concepts

Review

- Number system must include "Square root of negative numbers" for "completeness". This enables "closure" of the set of numbers.

- Square root of negative numbers was encountered while solving cubic equation for "Real" root. (Cardano 1545 AD)
- This problem was cleverly solved by Bombelli. (≅ 1560 AD)

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- In general we can represent square root of any negative number in terms of $\sqrt{-1}$

For Ex: $\sqrt{-16} = \sqrt{(-1)(16)}$

$= \sqrt{-1}\,.\sqrt{16}$

$= (\sqrt{-1}).4$

- $\sqrt{-1}$ was called an "Imaginary" Number by Rene DesCartes in 1637 AD. This term was coined as a derogatory term, since most people saw this number ($\sqrt{-1}$) as fictitious and useless!

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- DesCartes also created the complex number form ($a+b\sqrt{-1}$) — even though he disliked complex numbers! (where a - Real Part, $b\sqrt{-1}$ - Imaginary Part)
- Around 1747, Leonard Euler made significant contributions, which enabled "Imaginary" number as a useful and relevant number!

- He enabled visualisation of imaginary as a practically relevant number.

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- He is also credited with the formalisation of 'i' to represent $\sqrt{-1}$ and

Practical Relevance of 'i' (Euler)

- we have defined

$i = \sqrt{-1}$

Hence $i^2 = -1$

Note: $\sqrt{-1} = \pm i$ — we define $i = +\sqrt{-1}$!!

- Any positive number when multiplied by 'i^2' results in a negative number

for Ex: $1 i^2 = -1$

$4 i^2 = -4$ etc

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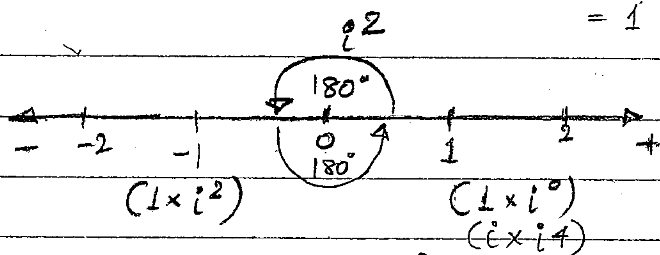
- Multiplication by i^2 can be represented graphically (geometrically!) as below:

For Ex $1 \times i^2 = -1$

$1 \times i^4 = 1 \times (-1) \times (-1) = 1$



- Geometrically, i^2 Rotates the given value by 180°!
- Similarly, i^4 rotates the given value by 360°!

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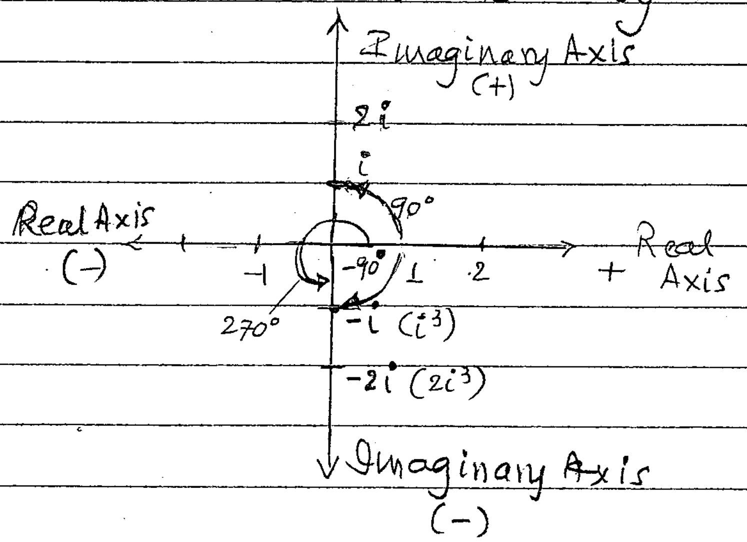
- Hence, we can visualise that multiplication by 'i' rotates the value by 90°



- Note that i^3 rotates the value by $90+90+90 = 270°$ or $-90°$

Also $i^3 = i^2 \cdot i = \underline{-i}$ (−90°!)

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- The above concept (or interpretation) essentially creates a <u>new</u> <u>dimension</u> for representing imaginary numbers.
- Hence, a complex number is a <u>number with two dimensions</u>!
- Infact, a <u>function of</u> <u>complex</u> <u>variable</u> can be represented as a graph in 3 dimensions (strictly speaking a graph in 4 dimensions!)

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- Even though, Euler contributed considerably in the area of complex numbers, he himself was not comfortable with negative numbers.
- Gauss (in early 1800's) formalised complex number $z = (a+bi)$ geometrically as a point (a, b) in the real 2-dimensional space!
- He also stated that mystery of negative and imaginary nos was due to ill adapted terminology.
- He suggested that <u>direct</u>, <u>inverse</u> and <u>lateral</u> would be preferable to positive, Negative & Imaginary!!