U3A Maths

3-Mar-2023 Term1/Week5

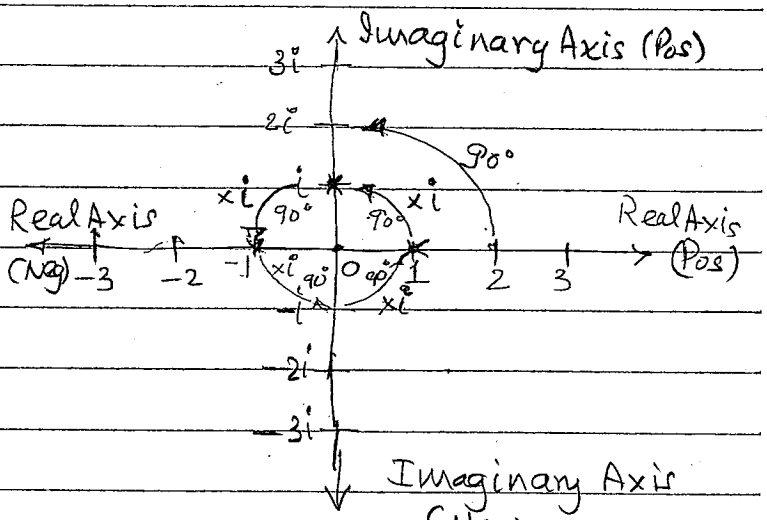
Complex Number Representation

Review

- Euler provided a method to "visualise" 'imaginary number (i)" in graphical form. (@1745 AD)

$i = \sqrt{-1}$ (By definition)

$i^0 = 1 \Rightarrow 0°$ - Real Axis positive
$i^1 = i \Rightarrow 90°$ - Imaginary Axis Positive
$i^2 = -1 \Rightarrow 180°$ - Real Axis Negative
$i^3 = -i \Rightarrow 270°$ - Imaginary Axis Negative
etc.



- Multiplication of a number by 'i' shifts the number by 90°

for Ex: $2 \times i = 2i$

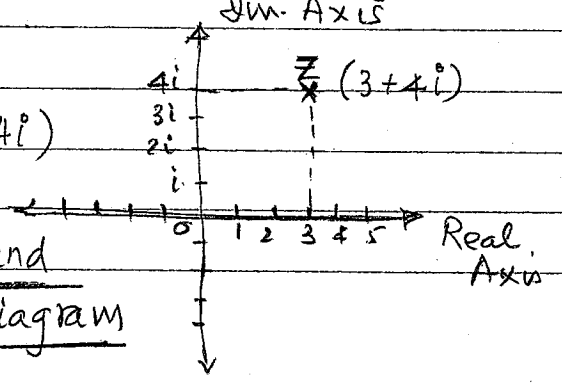
Real Axis Positive → Imaginary Axis (Positive)

- In early 1800s, Gauss formalised the Complex number as an algebraic expression

$Z = (a + bi)$

Real Part, Imaginary Part

- John-Robert Argand introduced representation of Complex numbers on a "two dimensional graph", as shown below



For Ex: $Z = (3+4i)$

Argand Diagram

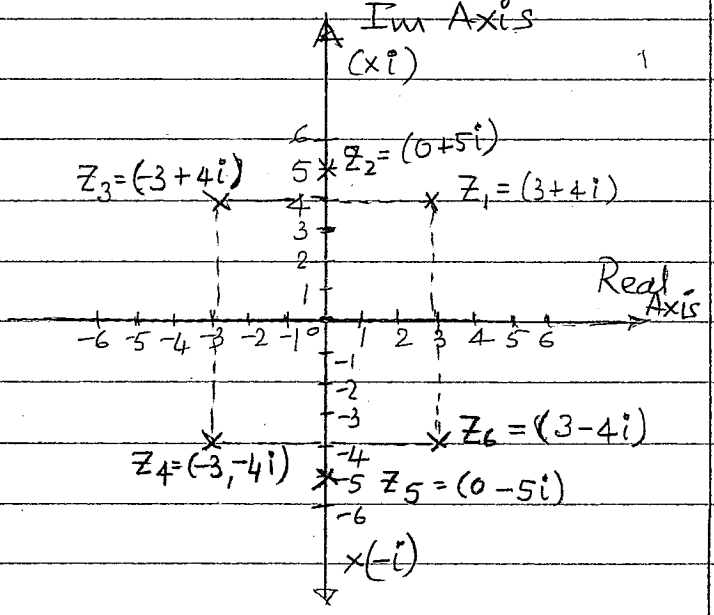
- Hence, we can think of a complex number as a point (number) on a 2-dimensional plane.

- The graphical representation of complex numbers is often referred to as Argand Diagram

Ex.1

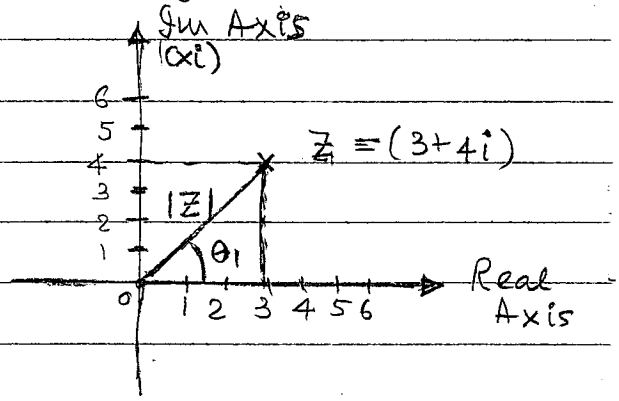
Represent the following complex numbers on a Argand Diagram.

$Z_1 = (3+4i)$, $Z_2 = (0+5i)$, $Z_3 = (-3+4i)$
$Z_4 = (-3-4i)$, $Z_5 = (0-5i)$, $Z_6 = (3-4i)$



- Note that a point on a two dimensional plane can also be located by
  - distance of the point to origin
  - Angle to Reference Axis (say, Real Pos. Axis)

Considering $Z = (3+4i)$



- The distance is normally referred to as the magnitude of complex number
  It is also called the modulus $\Rightarrow |Z|$
- The angle is referred to as the "angle of the complex number" (θ)

- We have, the distance from origin for 'Z'

$$\text{Mag. of } Z = |Z| = \sqrt{3^2+4^2} = 5$$

- The ange of 'Z'

$$= \theta = \tan^{-1}\left(\frac{4}{3}\right) = 53.13^\circ$$

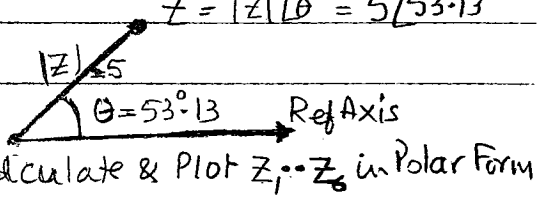
In general, we can write

for $Z = (a+bi)$

$$\text{Magnitude} = |Z| = \sqrt{a^2+b^2}$$

$$\text{Angle} = \theta = \tan^{-1}(b/a)$$

- Complex No. form: $Z = (a+bi)$ is called the Cartesean form
- Complex No. form $Z = |Z|\angle\theta$ is called the Polar form
- Note that for Polar form plot, we do not need the Real-Imaginary axes.
- All we need is a Reference point (origin) and Reference axis



Homework: Calculate & Plot $Z_1 \cdots Z_6$ in Polar Form