U3A Maths

24-Mon-2023 Term 1 / Week 8

Complex Number Forms

Who am I? (What am I?)

- That which is the subtle substance, all this (world) has as it's self; That is the Real, That is the self, You are That!, O'Svetaketu.

- Chandogya Upanishad (6.8.7.)
{c. 600 B.C.}

- A popular example of the Real, is the Gold Ring. Gold is the real substance,

where as, Ring has only (temporary) name & form. It can be converted into another name & form, for example, a Gold chain! Hence, Ring & Chain are not Real!

- It is interesting that, while explaining the behaviour of atoms (quantum physics), the equation contains the imaginary number 'i' ($\sqrt{-1}$)

$$H.\psi(r,t) = i\hbar \frac{\partial \psi}{\partial t}(r,t)$$

(Shrodënger's Equation).

Review

$e^{ix} = \cos(x) + i\sin(x)$

- Euler's Equation.

when $x = \pi$, $e^{i\pi} + 1 = 0$

- Euler's identity.

- e, i & π are transcendental numbers and 0 & 1 are the most fundamental numbers.

- Euler's identity is known as the most beautiful equation. It holds the Secret to the universe!

- We now three different forms for representing complex numbers.

- Cartesian Form

This is the most common form of a complex number, namely,

$$\mathbf{Z} = a + bi$$

where

a & b — Real numbers

i — imaginary number ($\sqrt{-1}$)

Z — Complex Number (normally bold font is used)

- This form is useful for addition & subtraction.

Ex: $Z_1 = (3+4i)$, $Z_2 = (2+3i)$

$$Z_1 + Z_2 = (3+4i) + (2+3i)$$
$$= \underline{(5+7i)}$$

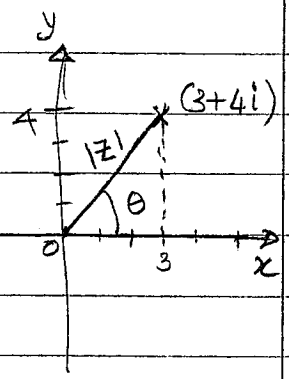
- In this form, a complex number is represented as a point on the x-y plane (Real-Imaginary axes). It is also called the Argand diagram.

Polar Form

- In practice, this is a popular form for representing the final result, in most practical applications.

- Here, the complex number is represented as a line (vector) at a given angle to the reference axis. The reference axis is normally the horizontal or the real ('x') axis on the Argand diagram.

Let $Z = (a+bi)$



then Polar form is

$$Z = |Z| \angle \theta$$

where $|Z| = \sqrt{a^2+b^2}$

$$\theta = \tan^{-1}(b/a)$$

For Ex: $Z = (3+4i)$

$$|Z| = \sqrt{3^2+4^2} = 5$$
$$\theta = \tan^{-1}(4/3) = 53.13 \text{ degrees}$$
or 0.927 radians

Note: π radians = 180 degrees
(3.1416 radians)

- Polar form is useful for multiplication & division

For Ex: $4 \angle 30^\circ \times 5 \angle 20^\circ$

$$= (4 \times 5) \angle 30^\circ + 20^\circ = \underline{20 \angle 50^\circ}$$

Exponential Form

- Complex number can be written as a formal algebraic expression using Euler's formula.

$$Z = |Z| e^{i\theta}$$
$$= |Z| (\cos\theta + i \sin\theta)$$
$$= |Z| \cos\theta + i |Z| \sin\theta$$

- Exponential form is useful for differentiation & integration!