

Complex Number Representation

Review

$Z = (a + j b)$ - Cartesian Form

$Z = |Z| \cdot e^{j\theta}$ - Exponential Form
 $= |Z| (\cos \theta + j \sin \theta)$

$Z = |Z| \angle \theta$ - Polar Form
 (simplified version of Exponential Form)

Note: No class in April!

Apr 7 - No class (Good Friday)

Apr 14 & 21 - No class (Term Break)

Apr 28 - No class (Sesha on Leave)

May 05 - class Restarts!

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(a) we have

$Z = |Z| \cdot e^{j\theta}$
 $= |Z| (\cos \theta + j \sin \theta)$

$\therefore |Z| \cos \theta + j |Z| \sin \theta = 3 + j 4$

$\therefore |Z| \cos \theta = 3$ - (1)

$|Z| \sin \theta = 4$ - (2)

$|Z|^2 \cdot \cos^2 \theta + |Z|^2 \sin^2 \theta = 3^2 + 4^2$
 $= 25$

$|Z|^2 (\cos^2 \theta + \sin^2 \theta) = 25$

$\therefore |Z|^2 = 25$

$|Z| = \sqrt{25} = 5$

we ignore negative value!

• Let us now consider a few examples.

Ex. 1

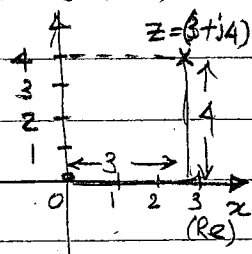
Plot $Z = (3 + 4j)$ on the Argand diagram and express 'Z' in the following forms

(a) Exponential Form

(b) Polar Form \angle (Sm)

(c) Plot the polar form

$Z = (3 + 4j)$



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Also, $\frac{(2)}{(1)} = \frac{|Z| \sin \theta}{|Z| \cos \theta} = \frac{4}{3}$

$\therefore \tan \theta = 0.75$

$\therefore \theta = \tan^{-1}(0.75)$
 $= \underline{36.87^\circ}$ or 0.644 radians

Hence,

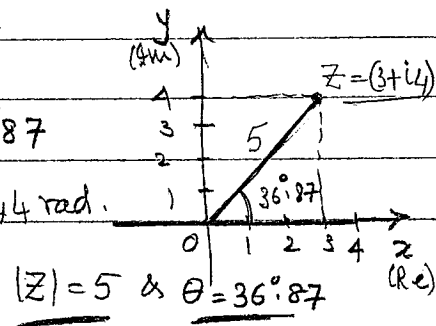
$Z = (3 + 4j) = \underline{5 \cdot e^{j 36.87^\circ}}$

or $\underline{5 \cdot e^{j 0.644}}$

(c) Polar Form

$Z = 5 \angle +36.87^\circ$

or $5 \angle 0.644 \text{ rad.}$



$|Z| = 5$ & $\theta = 36.87^\circ$

Ex. 2

Express $Z = 5 \angle 30^\circ$ in

(a) Exponential Form

(b) Cartesian Form

(c) Plot 'Z' in polar form

We have

$$Z = 5 \angle 30^\circ$$

(a) $Z = 5 \cdot e^{i30^\circ}$

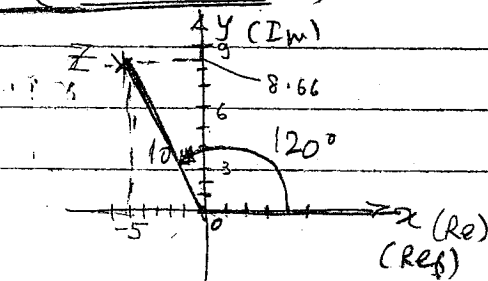
(b) $Z = 5 e^{i30^\circ}$
 $= 5 (\cos 30^\circ + i \sin 30^\circ)$
 $= 5 \cos 30^\circ + i 5 \sin 30^\circ$
 $= 5 \times 0.866 + i 5 \times 0.5$
 $= \underline{\underline{4.33 + i 2.5}}$

Ex. 3

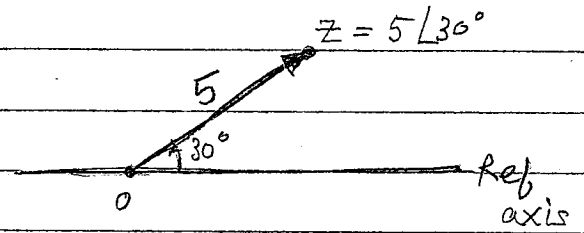
Express $Z = 10 \angle 120^\circ$ in Cartesian form & plot the value.

We have

$$Z = 10 \angle 120^\circ$$
$$= 10 e^{i120^\circ}$$
$$= 10 (\cos 120^\circ + i \sin 120^\circ)$$
$$= 10 \cos 120^\circ + i 10 \sin 120^\circ$$
$$= 10 \times (-0.5) + i 10 \times (+0.866)$$
$$Z = \underline{\underline{-5 + i 8.66}}$$



(c) Polar Form Plot



Note: (1) Polar form essentially represents a vector, since polar form has

- Magnitude
- Direction (represented by the angle to the reference.)

(2) Hence it can be used represent physical parameters such as, force, velocity etc.

Home Work

(1) Express the following in exponential & polar forms and plot the values

(a) $Z = (3 - i4)$

(b) $Z = (-3 - i4)$

(c) $Z = (240 + i0)$

(2) Express the following in Cartesian form and plot the values.

(a) $Z = 10 \angle -30^\circ$

(b) $Z = 20 \angle -120^\circ$

(c) $Z = 240 e^{i3\pi/2}$