U3A Maths

5-May-2023 Term2/Week2

Complex Numbers – Theory to Practice

- Imaginary number ($\sqrt{-1}$) was incorporated into the number system to consider square root of a negative number (for ex $\sqrt{-121}$). (Cardano, 1545AD)
- The square root of negative numbers were standardised by defining the imaginary number

$$i = \sqrt{-1}$$

- The above definition for 'i' enabled representation and manipulation (calculation!) of square root of negative numbers in a convenient way:

Ex: $\sqrt{-4} = \sqrt{-1}.\sqrt{4} = i2$

$\sqrt{-121} = \sqrt{-1}\sqrt{121} = i11$

etc

- The above resulted in the definition of a Complex number as

$$z = (x + iy)$$

- It can further be stated that a complex number is the "most general form" of a number!, since any arithmetic operations with complex numbers result in a complex number!
- In other words, what exists in reality (!) is a complex number, and the number system we use in practice (for day to day life), is essentially a particular case of the complex number!

- Euler ($\cong$ 1747 AD) gave an entirely new meaning to the imaginary number 'i'.
- $i = \sqrt{-1}$; $\therefore i^2 = -1$

Hence any number when multiplied by i^2, changes the sign (or direction!) of that number

Ex $4 \times i^2 = 4 \times -1$
$= -4$

- Hence, geometrically (graphically) this corresponds to the rotation of the number by 180°

- Hence, multiplication by 'i' rotates a given value by $90°$ - say in the anti-clockwise direction, which is our reference direction

- Euler also defined a general rotation of a given value by defining complex number as

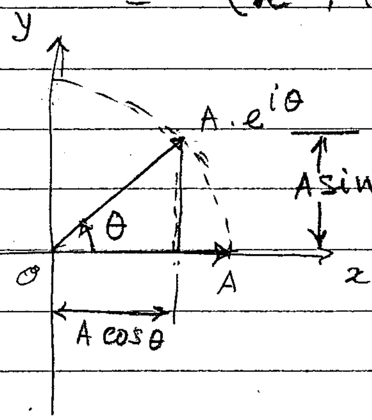
$$Z = A \cdot e^{i\theta}$$

where A — Magnitude
θ - Angle of rotation (in anti-clockwise)

- He also derived the identity
$$e^{i\theta} = \cos\theta + i\sin\theta$$

- Hence, we have,

$$Z = A e^{i\theta} = A(\cos\theta + i\sin\theta)$$
$$= \underbrace{A\cos\theta}_{} + \underbrace{iA\sin\theta}_{}$$
$$= (x + iy)$$



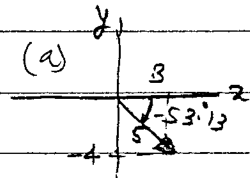
- Hence, a complex number can now be used to represent quantities which have magnitude and direction. It also provides an algebraic method to work with such quantities

- Practical examples of such quantities are Velocity, Force, AC voltage, AC current etc

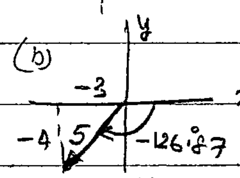
- In general, a complex number can be used to represent any "two dimensional quantity"

Home Work Solution (Ref: Term 1/Wk 9)

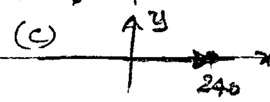
① (a) $(3-i4) = 5\angle{-53.13°}$
$= 5e^{-i53.13}$



(b) $(-3-i4) = 5\angle{-126.87}$
$= 5e^{-i126.87}$

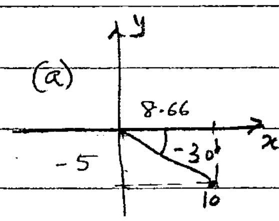


(c) $(240+i0) = 240\angle{0°}$
$= 240e^{i0}$

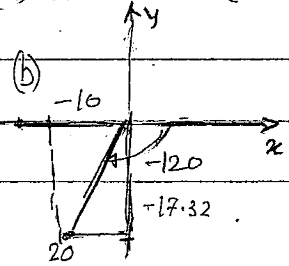


②

(a) $10\angle{-30°} = (8.66 - i5)$



(b) $20\angle{-120°} = (-10 - 17.32)$



(c) $240e^{i3\pi/2} = 240e^{i270}$
$= -i240$