U3A Maths

12-May-2023 Term 2 / Week 3

Complex Number Arithmetic

Review

Complex number:

$z = x + iy$ – Cartesian Form

$z = Ae^{i\theta}$ – Euler Form

where $e^{i\theta} = \cos\theta + i\sin\theta$

$(= \text{Cis}\,\theta) \Rightarrow$ Short form

$z = A\angle\theta$ – Polar Form

$= A(\cos\theta + i\sin\theta)$

Euler's form is very useful for practical applications, as it provides a method for representing Magnitude and

Direction by algebraic method. This provides a method for working physical quantities, such as, Force, Velocity, AC voltage etc with algebraic methodology.

Complex Number Arithmetic

Let us first consider cartesian form, which is versatile.

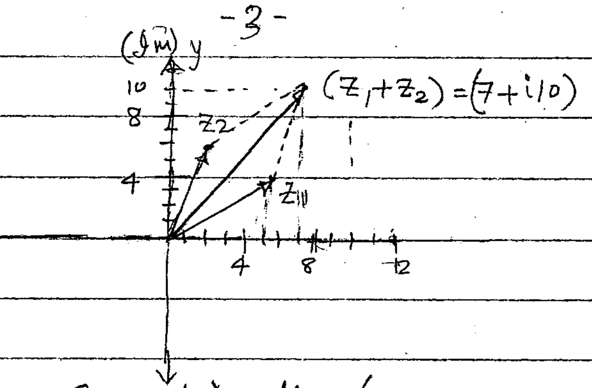
Addition

Ex.1 let $z_1 = (5+i4)$ & $z_2 = (2+i6)$

The sum is:

$(z_1 + z_2) = (5+i4) + (2+i6)$

$= (7+i10)$



Graphically $(z_1 + z_2)$ corresponds to the resultant of 2 forces z_1 & z_2.

- Complex numbers facilitate use of algebra, instead of geometry for such calculations.

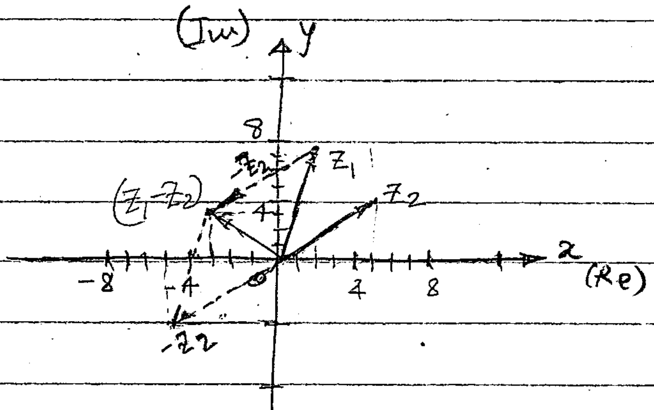
Subtraction

Ex.2 Let $z_1 = (2+i7)$ & $z_2 = (5+i4)$

Subtracting:

$(z_1 - z_2) = (2+i7) - (5+i4)$

$= (-3+i3)$



Note: $(z_1 - z_2) = z_1 + (-z_2)$

$= (2+i7) + (-5-i4)$

$= (-3+i3)$

Multiplication

Ex. 3 Let $z_1 = (3+i2)$ & $z_2 = (1+i4)$

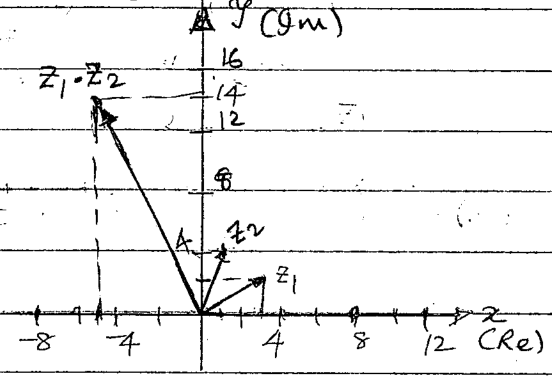
Multiplying, we have

$$z_1 \cdot z_2 = (3+i2)(1+i4)$$

We can do algebraic multiplication as usual, but remember that $i^2 = -1$

Let

$$z_3 = z_1 \cdot z_2 = (3+i2)(1+i4)$$
$$= 3 + i12 + i2 + i^2 8$$
$$= (3-8) + i(12+2)$$
$$= \underline{(-5+i14)}$$



The graph make more sense if the multiplication is done in Euler's or Polar form

Polar form:

$$z_1 = (3+i2) = \left(\sqrt{3^2+2^2}\right) \angle \tan^{-1}(2/3)$$
$$= 3.61 \angle 33.7^\circ$$

$$z_2 = (1+i4) = \left(\sqrt{1^2+4^2}\right) \angle \tan^{-1}(4/1)$$
$$= 4.12 \angle 76^\circ$$

Euler form

$$z_1 = 3.61\, e^{i33.7^\circ} \qquad z_2 = 4.12\, e^{i76^\circ}$$

$$\therefore z_1 \cdot z_2 = 3.61\, e^{i33.7^\circ} \times 4.12\, e^{i76^\circ}$$
$$= 14.87\, e^{i(33.7^\circ + 76^\circ)}$$
$$= 14.87\, e^{i109.7^\circ}$$
$$\text{or } \underline{14.87 \angle +109.7^\circ}$$

We also have

$$z_1 \cdot z_2 = (-5+i14) = \underline{14.87 \angle +109.7^\circ}$$

Verify the calculations Cartesean to Polar form!

Division

Let use use polar form!

Let $z_1 = (3+i2) = 3.61 \angle +33.7^\circ$

& $z_3 = (-5+i14) = 14.87 \angle +109.7^\circ$

We have

$$\frac{z_3}{z_1} = \frac{14.87 \angle 109.7^\circ}{3.61 \angle 33.7^\circ} = \underline{4.12 \angle 76^\circ}$$

This is z_2!

Home Work

Find z_3 / z_1 using Cartesean form

$$\frac{z_3}{z_1} = \frac{(-5+i14)}{(3+i2)} \times \frac{(3-i2)}{(3-i2)}$$

Note: use "conjugate" of the denominator $(3+i2)^* = \underline{(3-i2)}$