U3A Maths

19-May-2023 Term 2 / Week 4

Complex Number Arithmetic

- Addition & Subtraction of complex numbers are fairly straight forward in Cartesian form.

  For Ex: $(3+4i)+(5+6i)=(8+14i)$

- Convert the values into Cartesian form, if the data is given in Euler's or Polar form.

  Ex $5\angle +36^\circ 87 \Rightarrow (3+4i)$

  $7\cdot 81\angle +50^\circ 2 \Rightarrow (5+6i)$

- Addition / subtraction have direct applications in practice, for physical quantities with a magnitude & direction, such as, force, velocity, AC voltage etc.

Multiplication

- Multiplication of complex numbers follows the same procedure as in algebraic multiplication

For Ex:

let $Z_1=(3+i2)$ & $Z_2=(1+i4)$

The product:

$Z_1Z_2 = (3+i2)(1+i4)$

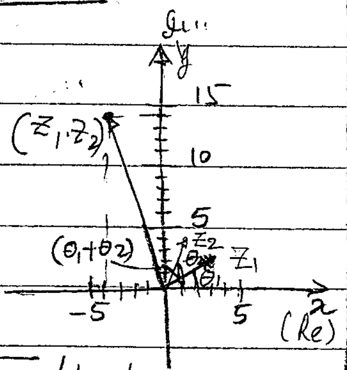
$= 3+i12+i2+i^2 8$

Note: $i^2=-1$

$= (3-8)+i(12+2)$

$= \underline{(-5+i14)}$

- The graphical form makes more sense in polar / Euler form multiplication.



$Z_1 = (3+i2) = \sqrt{3^2+2^2}\angle \tan^{-1}(2/3)$

$= 3\cdot 61\angle 33^\circ 7 = 3\cdot 61 e^{i33.7^\circ}$

$Z_2 = (1+i4) = 4\cdot 12\angle 76^\circ = 4\cdot 12 e^{i76^\circ}$

$Z_1 . Z_2 = 3\cdot 61 e^{i33.7^\circ} \times 4\cdot 12 e^{i76^\circ}$

$= 14\cdot 873\, e^{i(33^\circ 7+76^\circ)}$

$= 14\cdot 873\, e^{i109^\circ 7}$

$= 14\cdot 873\angle 109^\circ 7$

$= 14\cdot 873(\cos 109^\circ 7 + i\sin 109^\circ 7)$

$= \underline{(-5+i14)}$

- Polar / Euler form multiplication is simpler in practice:

  $Z_1 = A\angle\theta_1$; $Z_2 = B\angle\theta_2$

  $Z_1Z_2 = A\times B\angle(\theta_1+\theta_2)$

- Complex no. product does have practical use, but it needs some interpretation! For Ex:

A.C. Power = A.C. Voltage × AC Current
(??) (Complex) × (Complex)

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- In general, algebraic products have their own peculiarities,

for ex:

let $a \Rightarrow$ apples

$b \Rightarrow$ berries

$2 \times a \Rightarrow$ 2 apples ✓

$4 \times b \Rightarrow$ 4 berries ✓

We have $2a \times 4b = \underbrace{8\,ab}$

what does this signify?!

- Complex Constant × Complex Variable is okay!
- Complex variable × Complex Variable needs some interpretation!

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Complex Conjugate

- 'Conjugation' in general signifies changing of the sign, ie, + to − or − to +

- In the case of complex numbers, we define conjugation as changing the sign of the "imaginary part" or the "polar angle"

- Superscript (*) indicates conjugate.

$$Z = (x+iy) \Rightarrow Z^* = (x-iy)$$

or

$$Z = A\,\angle\theta \Rightarrow Z^* = A\,\angle{-\theta}$$

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Complex Division

- Complex division rarely has practical significance, but it is commonly encountered as a part of calculations in practical applications.
- complex division:

let $Z_1 = (3+i2)$ &

$Z_3 = (-5+i14)$

Calculate (Z_3/Z_1)

$$\frac{Z_3}{Z_1} = \frac{(-5+i14)}{(3+i2)} \times \frac{(3+i2)^*}{(3+i2)^*}$$

(Mult & divide by conjugate of the denominator)

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Use of conjugate helps to get a real number in the denominator!

$$\frac{Z_3}{Z_1} = \frac{(-5+i14)(3-i2)}{(3+i2)(3-i2)} = (1+i4)\ !!$$

- Division in polar form is much simpler,

let,

$Z_1 = (3+i2) = 3.61\angle 33.7^\circ$

$Z_3 = (-5+i14) = 14.873\angle 109.7^\circ$

$$\frac{Z_3}{Z_1} = \frac{14.873\angle 109.7^\circ}{3.61\angle 33.7} = 4.12\angle 109.7^\circ - 33.7^\circ$$

$$= 4.12\angle 76^\circ$$

- In polar form:

$Z_1 = A\angle\theta_1 \quad Z_2 = B\angle\theta_2$

$$\boxed{(Z_1/Z_2) = (A/B)\angle\theta_1 - \theta_2}$$