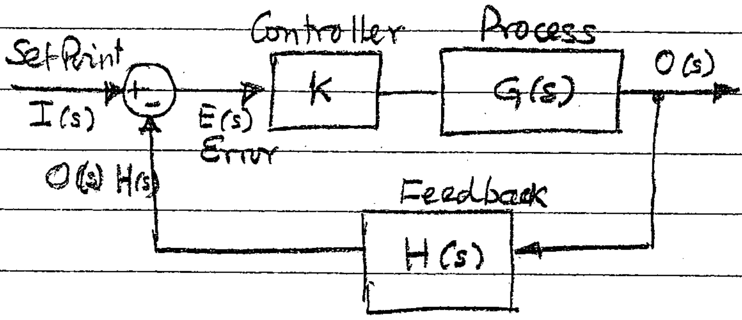
U3A Maths

16-Jun-2023 Term 2 / Week 8

Complex Roots & Control System Stability

Simple Control System



- In practice, G(s) & H(s) are differential equations, however, they are converted into algebraic equations

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using Laplace transformation

- 's' is the Laplace transformation variable
- By inspection of control system block diagram, we have,

$$E(s) = I(s) - O(s).H(s) \quad -(1)$$

$$O(s) = E(s)\,[K.G(s)] \quad -(2)$$

Eliminating E(s) & simplifying:

$$\frac{O(s)}{I(s)} = \frac{K.H(s).G(s)}{1+K\,H(s).G(s)} \quad -(3)$$

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- For stability analysis, we are interested in the roots of the denominator,

$$1 + K\,G(s)\,H(s) = 0$$

- For simplicity, let

$H(s) = 1$ ⇐ unity Feedback

the we have

$$\underline{1 + KG(s) = 0}$$

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Ex. 1 Given that

Process Eqn $G(s) = \dfrac{1}{s(s+1)(s+2)}$

and $K = 3$

Check whether the system is stable.

Substituting in $1+KG(s)=0$, we have;

$$1 + \frac{3}{s(s+1)(s+2)} = 0$$

$$\frac{s(s+1)(s+2)+3}{s(s+1)(s+2)} = 0$$

$$\therefore\ s(s+1)(s+2)+3 = 0$$

ie, $\underline{s^3 + 3s^2 + 2s + 3 = 0}$

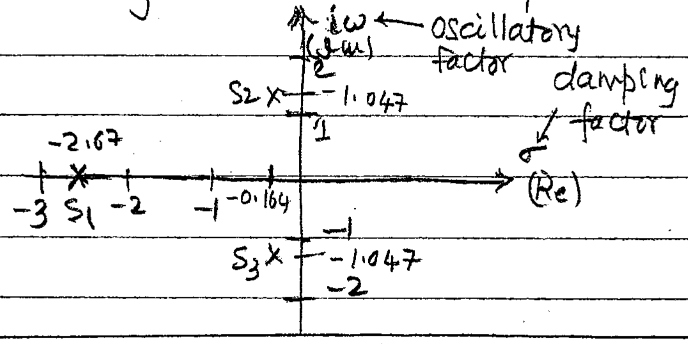
- we need to find the roots of the above cubic equation!

- let us use the tools available on the web site - "subjectcoach.com"!
- Search for "Roots of Cubic Equation"
- $S_1 = -2.672$

  $S_2 = -0.164 + i\,1.047$

  $S_3 = -0.164 - i\,1.047$

Plotting the roots on 's' plane.



Conditions for Stability

- Real part of the roots must be negative
- Oscillations are present if there are imaginary parts. (the system is stable if the real part is negative)
- The system is unstable if the real part is positive.
- The magnitudes of real & imaginary parts correspond the magnitude of damping and oscillations respectively.

Conclusion:

The system is oscillatory but stable.

Ex. 2 Given that,

$$G(s) = \frac{1}{s(s+1)(s+2)}$$

and $K = 7$

check whether the control system is stable.

we have:

$$1 + KG(s) = 0$$

$$1 + \frac{7}{s(s+1)(s+2)} = 0$$

Simplifying we get.

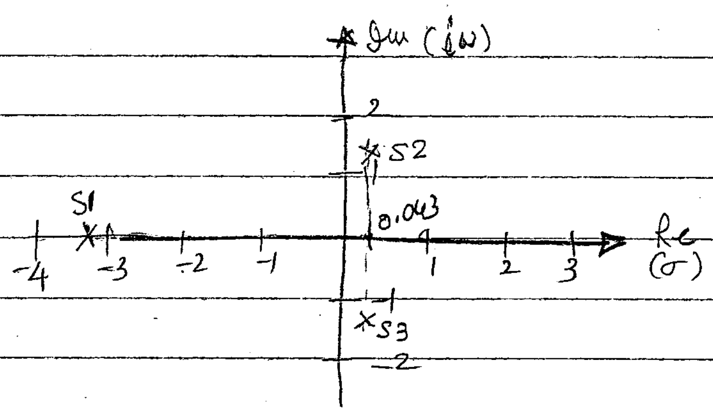
$$s^3 + 3s^2 + 2s + 7 = 0$$

- The roots of the equation are

  $S_1 = -3.086$

  $S_2 = 0.043 + i\,1.505$

  $S_3 = 0.043 - i\,1.505$



- The system is oscillatory and unstable!