U3A Maths

23-Jun-2023 Term 2 / Week 9

Fourier Transforms

- Fourier Transform is the most used mathematical equation in modern day life!
- For Ex, Every time a picture is sent over the internet, (Fast) Fourier Transform (FFT) is used!
- The concept was first introduced by Joseph Fourier (1768-1830) - a French mathematician.
- Apparently, Gauss had

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developed the FFT, but he never published it!

- In practice, mathematics is well equipped to analyse parameters (physical quantities) which vary as per standard functions, such as, linear, cubic, sinusoidal, exponential etc.
- However, analysis of systems with non-standard or random variations is not feasible with classical maths.

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- Joseph Fourier, while studying flow of heat in solid bodies, came up with a smart discovery.

"Any function of a variable can be expanded (resolved) in a series of multiples of sine functions of that variable".

- The above is called "Fourier Series", which was later generalised and called "Fourier Transform".

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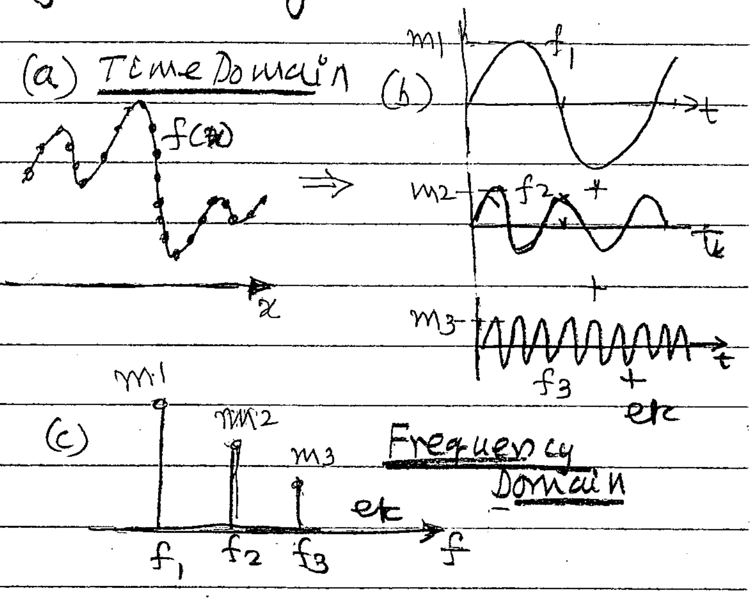
- This can be illustrated graphically as below:



- Note that figure (c) data are much easier to transmit, ie, (f_1, m_1), (f_2, m_2) etc compared with figure (a)

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- Frequcy & Magnitude transmission is very efficient to store & transmit, especially since frequencies with "relatively small" magnitude need not be stored or transmitted. However, signal or picture quality is still surprisingly close to the original!

- In practice, efficiencies of about 100:1 can be achieved!

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- For Ex: a picture with 1 MegaBits (1,000,000 bits) can be reduced to 1 MegaBits / 100 = 10 kB (10,000 bits)!

- Practical applications of Fourier Transforms include:
  - Signal processing (sound)
  - Image processing
  - Noise Filtering
  - Ambient noise cancelling head phones
  - Spectral (frequency) analysis – vibrations, earth quake, etc.

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- Fourier transform is mathematically defined as:

$$F(\omega) = \int_{-\infty}^{+\infty} f(t) \cdot e^{-j\omega t} \, dt$$

where $\omega = 2\pi f$ (angular freq.)

'f' is the number of sinusoidal cycles/sec.

$f(t) \Rightarrow$ Function of time { Raw Variations of the variable in real life }

$F(\omega) \Rightarrow$ Function of frequency { Transformed Variations in frequency domain }

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- Inverse Fourier Transform is used to get back the original function (signal), as below:

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) \cdot e^{+j\omega t} \, d\omega$$

- In the present (digital) world, the above equations are rarely used!

- Instead, we use a much simpler equation called the "Digital Fourier Transform" (DFT).