U3A Maths.

28-JUL-2013 Term 3 / Week 2

Complex Numbers - An Alternate View
(© Sesha!)

- Introduction of imaginary variable ($i = \sqrt{-1}$) as a part of complex numbers, has been an eternal source of confusion.

- Here an alternative way to introduce imaginary variable ($i = \sqrt{-1}$) is presented. Hopefully, it will help to clear the concepts.

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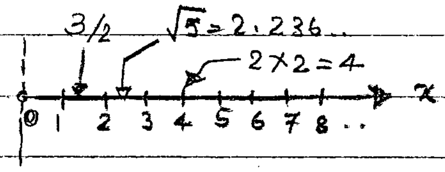
- When "natural" numbers were developed, they were essentially positive numbers
- One could perform addition, multiplication and division and the result is always a positive number.
- We can graphically represent positive numbers as below:



- We encounter a problem in the case of subtraction
For ex, $5 - 2 = 3$ ✓ $2 - 5 = ?$

-3-

- In fact, the negative numbers were not accepted in the Western world till 1500's (AD)
- It is difficult to concieve a negative number in practice, For ex: -4 apples
- However, "subtraction" ($2 - 5 = -3$) results in a negative number, hence, mathematics has to consider it for the sake of complete ness,

For Ex: $2 - 5 + 8 = -3 + 8$
$= \underline{+5}$

- The concept of negative number is required - since the final result can be positive!

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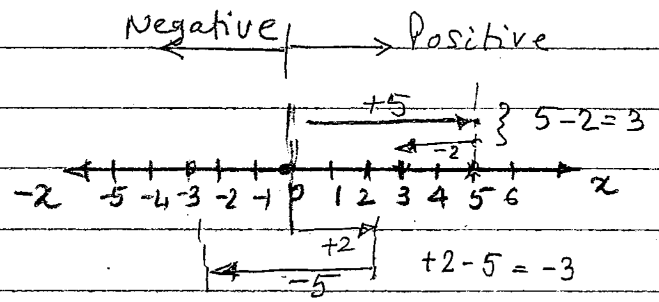
- We can now think of The negative numbers graphically as the "direction" of the number.



- Hence, the negative number can be considered to be in the "opposite" direction to the assumed positive direction.

- The above concept is useful for addition/subtraction, but is confusing for multiplication.

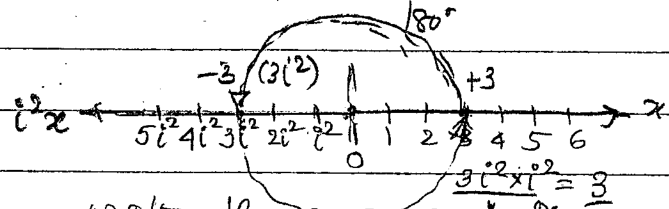
- For Ex: $+3 \times +5 = +15$

  $-3 \times -5 = \underline{+15}!$ (not -15)

- $- \times - \Rightarrow +$ is not logical and hence confusing.

- Let us define $i^2 = -1$ and corresponds to rotation of a given value by $180°$. (opposite direction)
- In other words, let us replace "negative sign" by i^2.

- Rewriting the diagram we have,



- With the new notation, we can write,

  for Ex: $4 - 3 \Rightarrow (4 + 3i^2)$

  $= 4 - 3 = 1$

  (where $i^2 = -1$)

- Let us now consider multiplication,

  For Ex: $-4 \times -3 = 4i^2 \times 3i^2$

  $= 12\, i^4$

  $= \underline{+12}!$

  Note that $i^4 = i^2 \times i^2 \Rightarrow$ the value is rotated by $360°$!

- Finally, for completeness, we will need to consider square root of a negative number.
- For Ex: $\sqrt{-16} = \sqrt{(-1)(16)}$

  $= \sqrt{-1} \times \sqrt{16}$

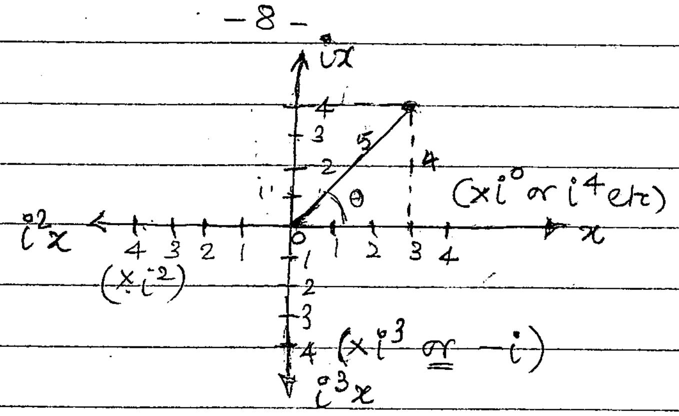
  $= \sqrt{i^2} \times 4$

  $= \underline{4i}$

- Hence we can write,

  $3 + \sqrt{-16} = (3 + 4i)$

- With the above feature we can perform calculations involving square root of negative numbers.
- Graphically $i = \sqrt{-1}$, represents rotation of the value by $90°$ anticlockwise.



- Hence, mathematically the most general form of number is a "complex" number

- Euler's equation provides for the most general form of a number

  $$A e^{i\theta} = A(\cos\theta + i\sin\theta)$$

For Ex:

$5e^{i0} = 5$ (Pos. No.); $5e^{i\pi} = -5$ (Neg No); $5e^{i\pi/2} = 5i$ (Sq. Root of Neg. no)