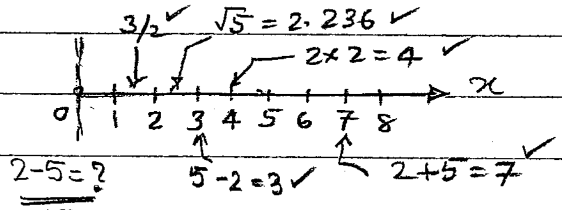
U3A Maths

4-Aug-2023 Term 3/Week 3

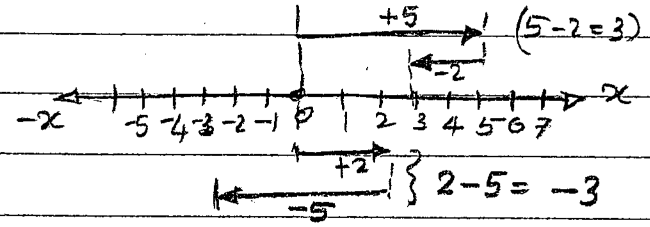
D.F.T. Calculations

- Complex Numbers - An Alternate View
  Review
  - Set of positive numbers is "Incomplete" for subtraction



- For Ex: $5-2=3$ is okay! however $(2-5)$ results in a number which is not a part of "set of positive numbers"

- A "negative" number can be associated with "direction"

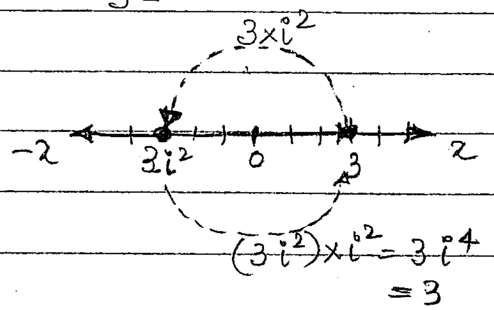


- For mathematical completeness it is better to represent negative numbers using $i^2 = -1$

For Ex: $2-5 = (2+5i^2)$

- The mathematical operation "i^2" shifts the value by 180°, in the anticlockwise direction

- For Ex



$i^4 \Rightarrow$ represents a rotation of 360°

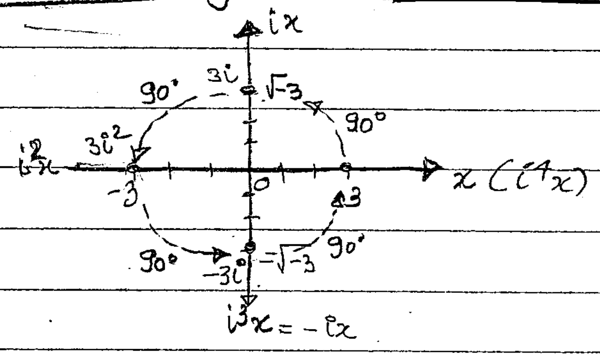
- The above represents pos. and neg. numbers. However, for completeness, we also need to represent "square root for negative" numbers

- We have $i = \sqrt{i^2} = \sqrt{-1}$ to represent such numbers.

For Ex: $\sqrt{-16} = \sqrt{(-1)(16)}$

$= \sqrt{(-1)} \cdot \sqrt{16} = i4$ or $4i$

- Similar to "i^2", we can now think that "i" shifts (rotates) the value by 90° anticlockwise.



- The most general form of the number can be written as:

$$Ae^{i\theta} = A\cos\theta + iA\sin\theta.$$

Ex: $5e^{i0^\circ} = 5(\cos 0^\circ + i\sin 0^\circ) = 5$ (Pos. No.)

$5e^{i\pi} = 5(\cos\pi + i\sin\pi) = -5$ (Neg. No.)

$5e^{i\pi/2} = 5(\cos\pi/2 + i\sin\pi/2) = 5i$ or $\sqrt{-5}$

$5e^{i53.13^\circ} = 5(\cos 53.13^\circ + i\sin 53.13^\circ) = (3+i4)$ or $(3+\sqrt{-16})$

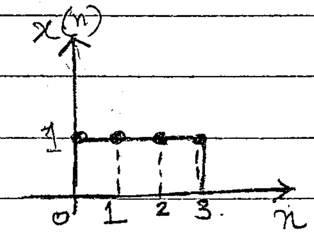
General Form of Number!

D.F.T. Home work Example

Given that

$x = \{1\ 1\ 1\ 1\} \Rightarrow \{x(0), x(1), x(2), x(3)\}$

find D.F.T. and consequently inverse D.F.T..



we have 4 values

$\therefore N = 4$ & $(N-1) = 3$

(In practice, N Corresponds to total "time period" (T) of one cycle or 2π radians)

D.F.T Equation: $F(K) = \sum_{n=0}^{(N-1)} x(n)\, e^{-\frac{i 2\pi k n}{N}}$ for $K = 0$ to $(N-1)$

For K=0

$$F(0) = \sum_{n=0}^{3} x(n)\, e^{-\frac{i 2\pi . 0 . n}{4}}$$

$$= x(0) e^{-i0} + x(1) e^{-i0} + x(2) e^{-i0} + x(3) e^{-i0}$$

$$= 1+1+1+1 = \underline{4} \quad (\text{ie, } F(0) = 4)$$

For k=1

$$F(1) = x(0). e^{-\frac{i 2\pi .1.0}{4}} + x(1) e^{-\frac{i 2\pi .1.1}{4}} + x(2) e^{-\frac{i 2\pi .1.2}{4}} + x(3) e^{-\frac{i 2\pi .1.3}{4}}$$

$$= 1. e^{-i0} + 1. e^{-i\pi/2} + 1. e^{-i\pi} + 1. e^{-\frac{i 3\pi}{2}}$$

$$= 1 + 1. (\cos\tfrac{\pi}{2} - i \sin\tfrac{\pi}{2}) + 1. (\cos\pi - i\sin\pi) + 1. (\cos\tfrac{3\pi}{2} - i\sin\tfrac{3\pi}{2})$$

$$= 1 + (0 - i1) + (-1 - 0i) + [0 - (-i1)]$$

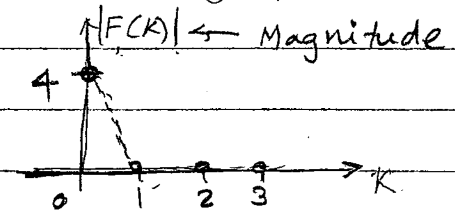
$$= \cancel{1} - \cancel{i1} - \cancel{1} + \cancel{i1} = \underline{0}$$

$$\therefore \underline{F(1) = 0}$$

Similarly, we can show that

$\underline{F(2) = 0}$ & $\underline{F(3) = 0}$

The D.F.T. in graphical form:



Note: In general, $x(0), x(1), \ldots$ are Real values & $F(0), F(1), \ldots$ are Complex values

Inverse D.F.T equation

$$x(n) = \frac{1}{N} \sum_{k=0}^{(N-1)} F(k) . e^{+\frac{i 2\pi n k}{N}}$$

for $n = 0$ to $(N-1)$

For our example,

$N = 4$ & $(N-1) = 3$ &

$F(0) = 1, \; F(1) = 0, \; F(2) = 0, \; F(3) = 0$

We have to find $x(0), x(1), x(2), x(3)$ $(n = 0, 1, 2, 3)$

For n=0

$$x(0) = \frac{1}{4} \sum_{K=0}^{3} F(K)\, e^{+\frac{i 2\pi . 0 . K}{4}}$$

$$= \tfrac{1}{4} [F(0) e^{i0} + F(1) e^{i0} + F(2) e^{i0} + F(3) e^{i0}]$$

$$= \tfrac{1}{4} [4 + 0 + 0 + 0] = \underline{1}$$

For n=1

$$x(1) = \frac{1}{4} \sum_{K=0}^{3} F(K)\, e^{\frac{i 2\pi .1.K}{4}}$$

$$= \tfrac{1}{4} \left[F(0). e^{\frac{i 2\pi .1.0}{4}} + F(1). e^{\frac{i 2\pi .1.1}{4}} + F(2)\, e^{\frac{i 2\pi .1.2}{4}} + F(3)\, e^{\frac{i 2\pi .1.3}{4}}\right]$$

$$= \tfrac{1}{4} [4 + 0 + 0 + 0] = \underline{1}$$

Similarly we can show

$\underline{x(2) = 1}$ & $\underline{x(3) = 1}$