U3A Maths

25-Aug-2023 Term 3 / Week 6

Complex Function Plotting

- Plotting of an (algebraic) function provides a visual image of the function.
- As humans are visual animals, plots help to provide a better feel for an unintelligible algebraic expression.
- Note: Humans are very good in pattern recognition, where as computers are good with algebraic expressions!!

- Let us first consider traditional algebraic function. For Ex,

$$f(x) = x^2 + 1$$

- We plot such functions using perpendicular axes in two dimensions. Such a system is called "Cartesian coordinate" system.
- It is named after Rene DesCartes (1595-1650 AD) - a French mathematician and philosopher. (Cartes ⇒ Cartesian!)

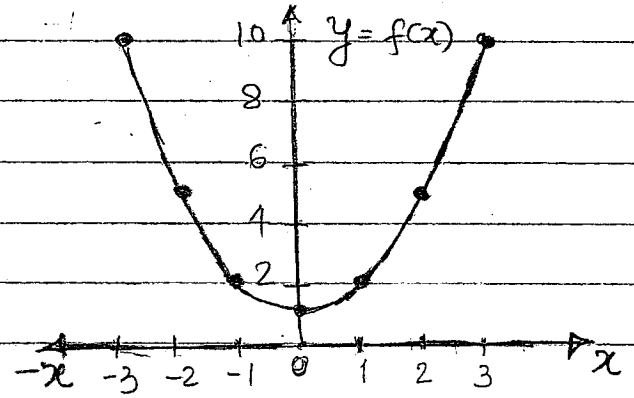
- This invention by DesCartes helped to relate Geometry with Algebra!
- Traditionally, "variable" value is plotted along horizontal axis and is called the 'x'-axis and the "function" value is plotted along the vertical axis and is called the 'y'-axis.
- Hence, we often write

$$y = f(x) = x^2 + 1$$

- Use of "y" is often confusing, since vertical axis essentially function (f(x)) values.

- Let us plot the function values for $x = -3$ to $+3$ in steps of 1

$x \Rightarrow$	-3	-2	-1	0	1	2	3
$f(x) \Rightarrow$	10	5	2	1	2	5	10



- Note that Cartesian system works only in 2 dimensions.

- Let us now consider the same function with a complex variable, say $z = (x+iy)$

[Do not confuse 'y' here with function $f(x)$. They have no relation - even though we will be using 'x' and 'y' as coordinate axes later!]

We have: the complex function

$$f(z) = z^2 + 1$$

where $z = (x+iy)$ is the complex variable. 'x' & 'y' can take any numerical (real) value.

- Obviously, in general, $f(z)$ yields a complex number. Let us say, such a value is

$$w = (u+iv) = f(z) = z^2+1$$

where w is a complex number
u & v are real numbers.

- Let us calculate the function values for $x = -2$ to $+2$ and to keep it simple let $y = 1$.

	$z = (x + iy)$		$w = (u + iv)$	
①	-2	1	4	-4
②	-1	1	1	-2
③	0	1	0	0
④	1	1	1	2
⑤	2	1	4	4

- To plot complex functions, we need a 4-dimensional coordinate system! Our cartesian coordinate system is not suitable!

- One alternative is to plot (x, y & u) and (x, y & v) as 3 dimensional plots. Of course, we will have two such plots, one for the real part (u) & another for the imaginary part (v) (we will explore this later)

- Another interesting alternative is to use 2 cartesian plots, one for 'Input' [$z = (x+iy)$] and one for 'Output' [$w = (u+iv)$]