19-Sep-2023   Term 3 / Week 10

Joukowsky Transformation
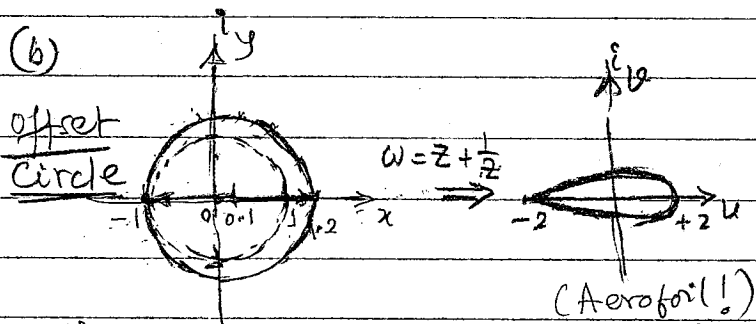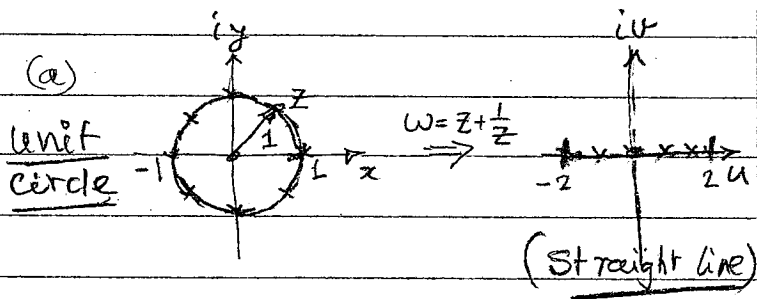### (Aerofoil Design)

- The complex function

$$w = f(z) = z + \frac{1}{z}$$

which we used for the example last week is called "Joukowsky Equation"

- It is popularly used for air plane wing design

- Nikolay Zhu_kovsky (1847-1921)
  (!)

- Founder of aero & hydro dynamics.
- One of the first to concieve human flight!
- first person to mathematically explain "aerodynamic lift" & built wind tunnel in Russia
- He also developed "water hammer" equation!

- Last week's example illustrated that Joukowsky equation converts a "circle" into "ellipse", when the radius = 2.

- More interesting shapes can be obtained by considering a 'unit' circle.

(a)
unit circle

$w = z + \frac{1}{z}$

(Straight line)

(b)
offset circle

$w = z + \frac{1}{z}$
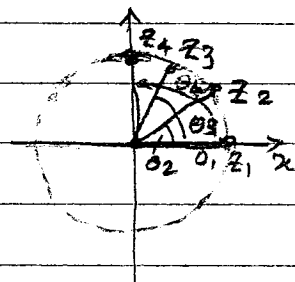
(Aerofoil!)

centre offset = +0.1

Circle radius = 1 + 0.1 = 1.1
                    ↑
                  offset

- For obtaing 'good' plots we need have atleast about 16 points on the circle.

- One convenient way to generate the points is to polar form of complex numbers.

For Ex:

Assuming mag or radius = 1

$z_1 = 1 \angle 0° = (1 + i\,0)$

$z_2 = 1 \angle +30° = (\cos 30° + i \sin 30°) = (0.867 + i\,0.5)$

$z_3 = 1 \angle +60° = (\cos 60° + i \sin 60°) = (0.5 + i\,0.867)$
                                                                etc.

- Next page gives a simple 'Octave' program to generate aerofoil shapes!

Joukowsky Function – Aerofoil Design

```
inc = 16;                                # specify the angle increment
th = [0 : pi/inc : 2*pi];                # generate theta values to form a circle
N = 2 * inc + 1;                         # Note: The above generates (2*inc+1) values
r = 1;                                   # specify radius for 'reference' circle
offset = complex(-0.1,0.15);             # specify offset (complex number makes it general)
R = r + abs(offset);                     # radius for 'offset' circle


z0 = complex( r.*cos(th), r.*sin(th) );  # generate values for points (z0) on reference circle
z = complex( R.*cos(th), R.*sin(th) );   # generate values for points (z) on offset circle


z = z + offset;                          # shift the circle as per the specified offset


zx = 1 ./ z;                             # calculate Joukowsky function w = f(z) = z + 1/z
w = z .+ zx;                             # (Note: '.' Specifies operation on each element of the array)


% Plot all results
figure 1;
axis([-3,3,-3,3]);
hold on;
plot(z0, '-.');                          # plot reference circle  (dotted circle in the Figure 1)
plot(z, '-*');                           # plot offset circle  (starred circle in Figure 1)
plot(offset, 'x');                       # plot offset (centre for offset circle, shown as 'x')


figure 2;
axis([-3 3 -3 3]);
hold on
plot(w, '-x');                           # plot Joukowsky function (aerofoil)  (Figure 2)
```
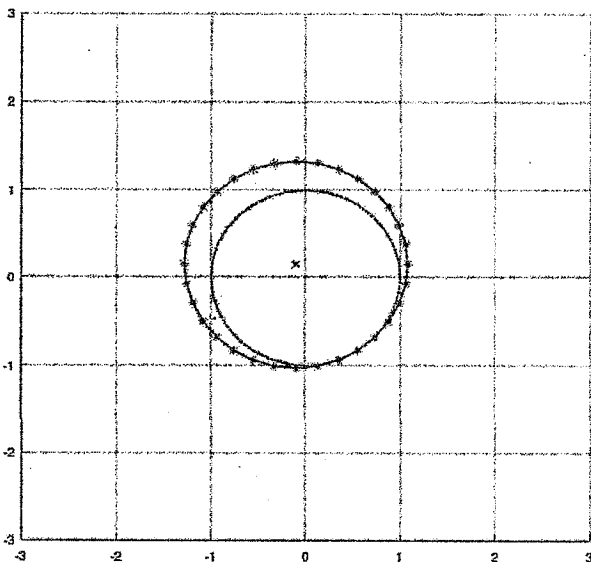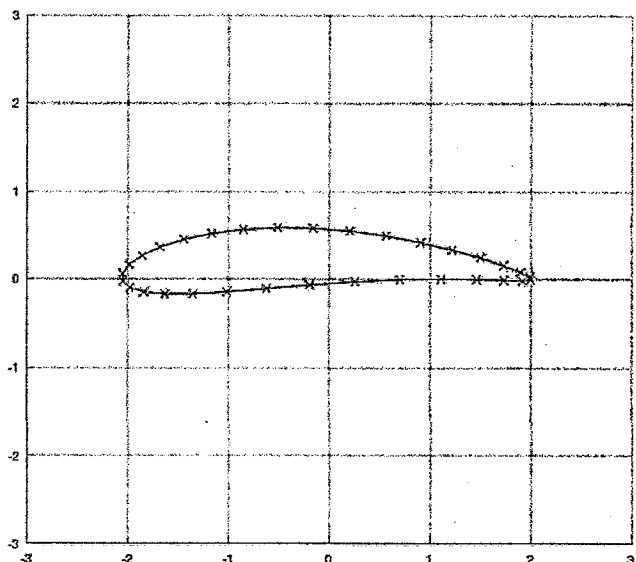

Figure 1

Figure 2