U3A Maths

25-Aug-2023 Term 3 / Week 7

Complex Function Plots

- Last week we had an intro to complex function plotting.
- Complex function plots require 4 dimensions, but we have only 3 dimensions!
- Hence, Complex function plots are done separately for Real part, Imaginary Part, Magnitude or phase angle.

- A 3 dimensional plot results in a surface - not a line. Hence, it is not practical to plot in 3 dimensions without a computing aid. (We will revisit this at a later date.)
- There is another interesting way to plot complex functions - which is relevant for 4 dimensions!!
- Such a plot is called a "Map"

- Mapping of complex functions with some particular features (not discussed here for simplicity) is called "conformal (function) mapping"
- Conformal mapping has very useful practical applications!
- For the present let us consider examples of complex function mapping and practice them.

We can write a complex function as below:

$$w = f(z)$$

where z is the complex variable, say $z = x + iy$

(Note the x & y are "input" values.

Note that the "output" of [complex function] $f(z)$ is also a complex value, let us say

$$w = (u + iv) = f(z)$$

Note: For "simple" variable we write $y = f(x)$ (output, input)

Ex: For complex function

$w = f(z) = z^2$

Calculate the function values for

$z_1 = (x_1 + iy_1) = (-2 + i1)$

$z_2 = (x_2 + iy_2) = (-1 + i1)$

$z_3 = (x_3 + iy_3) = (0 + i1)$

$z_4 = (x_4 + iy_4) = (1 + i1)$

$z_5 = (x_5 + iy_5) = (2 + i1)$

If you are confused, ignore x & y and just use the complex numbers for calculations.

we have $z_1 = (-2 + i1)$

$\therefore f(z_1) = z_1^2 = (-2 + i1)^2$

$= (-2)^2 + 2(-2)(i1) + (i1)^2$

$= 4 - i4 - 1$

$= (3 - i4)$

$\therefore w_1 = (u_1 + iv_1) = f(z_1)$

$= (3 - i4)$

Similarly,

$f(z_2) = z_2^2 = (-1 + i1)^2$

$= (-1)^2 + 2(-1)(i1) + (i1)^2$

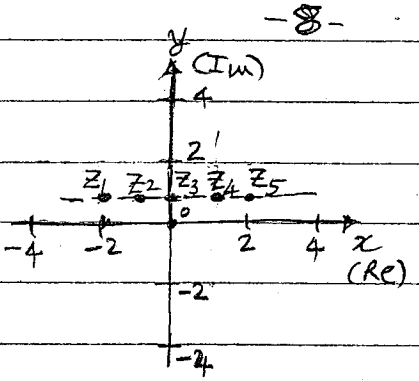
$= 1 - 2i - 1$

$= (0 - i2) = (u_2 + iv_2)$

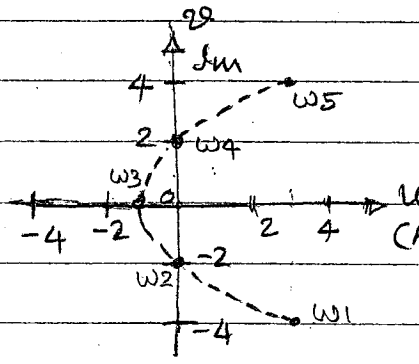
Continuing the calculations, we can list the results as below:

z $(x + iy)$	$f(z)$ w $(u + iv)$
$z_1 = (-2 + i1)$	$w_1 = (3 - i4)$
$z_2 = (-1 + i1)$	$w_2 = (0 - i2)$
$z_3 = (0 + i1)$	$w_3 = (-1 + i0)$
$z_4 = (1 + i1)$	$w_4 = (0 + i2)$
$z_5 = (2 + i1)$	$w_5 = (3 + i4)$

Let us plot 'z' values on (x, y) plane and 'w' values on (u, v) plane



Input Plot (z values)



Output Plot (w or f(z) values)

Home work Plot the following values (z & w) on the above graph.

$z_1 = (1 - i2)$; $z_2 = (1 - i1)$; $z_3 = (1 + i0)$

$z_4 = (1 + i1)$; $z_5 = (1 + i2)$