U3A Maths

12-Sep-2023 Term 3 / Week 9

Complex Function Plots (Cont'd)

We can write a complex function as below:

$$\omega = f(z)$$

wher $z = (x+iy)$ - Complex No.

$\omega = (u+iv)$ - Complex No.

- Using Cartesian form

$$\underbrace{(u+iv)}_{\omega} = f\underbrace{(x+iy)}_{z}$$

- We need 4 axes to plot complex functions, namely,

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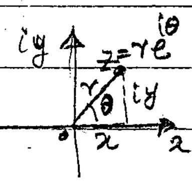
x & y $\Rightarrow$ 2 input values (axes)

u & v $\Rightarrow$ 2 output values (axes)



- Using Euler's form:

let $z = re^{i\theta} = (\underbrace{r\cos\theta}_{x} + i\underbrace{r\sin\theta}_{y})$

$\omega = R.e^{i\phi} = (\underbrace{R\cos\phi}_{u} + i\underbrace{R\sin\phi}_{v})$

$\therefore$ $\omega = f(z)$ can be written as

$$Re^{i\theta} = f(re^{i\theta})$$

- Again, we have 2 input values (r, θ) & 2 output values (R, ϕ) for a Polar Plot.

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- However, in this "world" we have only 3 dimensions, hence, only 3 axes are possible
- So, we generally plot 2 input values (x & y) and 1 output value (u, v, R or ϕ)!
- The above is called 3-dimensional plots and are popular in practice.
- Another interesting way is to plot input & output values on 2 separate figures or charts.
- In such a case, we can have 4 axes!!

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- Such a complex fuction plot is called (complex function) Conformal Map.

Note

[- There are some requirements to call "Conformal" Map, but let us not worry about it!]

Homework

Plot $\omega = f(z) = z^2$ using conformal maps for input values

$z_1 = (1-i2)$; $z_2 = (1-i1)$

$z_3 = (1+i0)$; $z_4 = (1+i1)$

$z_5 = (1+i2)$

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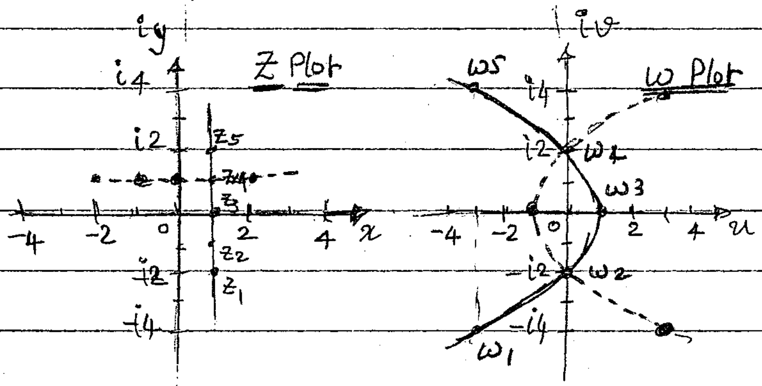
We have,

z $(x+iy)$	$w = f(z) = z^2$ w $(u+iv)$
z_1: $(1-i2)$	$w1$: $(-3-i4)$
z_2: $(1-i1)$	$w2$: $(0-i2)$
z_3: $(1+i0)$	$w3$: $(1+i0)$
z_4: $(1+i1)$	$w4$: $(0+i2)$
z_5: $(1+i2)$	$w5$: $(-3+i4)$



Note: The dotted lines are for last week's example!

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- The real power of conformal mapping is illustrated by the following example!

Ex.1

For the complex function

$$w = f(z) = \left(z + \frac{1}{z}\right)$$

Plot the z-w map for the following input values.

$z_1 = (2+0i)$ $\quad$ $z_5 = (-2+0i)$

$z_2 = (1.414+i1.414)$ $\quad$ $z_6 = (-1.414-i1.414)$

$z_3 = (0+i2)$ $\quad$ $z_7 = (0-i2)$

$z_4 = (-1.414+i1.414)$ $\quad$ $z_8 = (1.414-i1.414)$

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The results are:

z $(x+iy)$		$w = (z + 1/z)$ $(u+iv)$	
$z1$	$(2+0i)$	$w1$	$(2.5+0i)$
$z2$	$(1.414+i1.414)$	$w2$	$(1.768+i1.06)$
$z3$	$(0+i2)$	$w3$	$(0+i1.5)$
$z4$	$(-1.414+i1.414)$	$w4$	$(-1.768+i1.06)$
$z5$	$(-2+0i)$	$w5$	$(-2.5+0i)$
$z6$	$(-1.414-i1.414)$	$w6$	$(-1.768-i1.06)$
$z7$	$(0-i2)$	$w7$	$(0-i1.5)$
$z8$	$(1.414-i1.414)$	$w8$	$(1.768-i1.06)$

Note: z-map $\Rightarrow$ Circle

w-map $\Rightarrow$ Ellipse!

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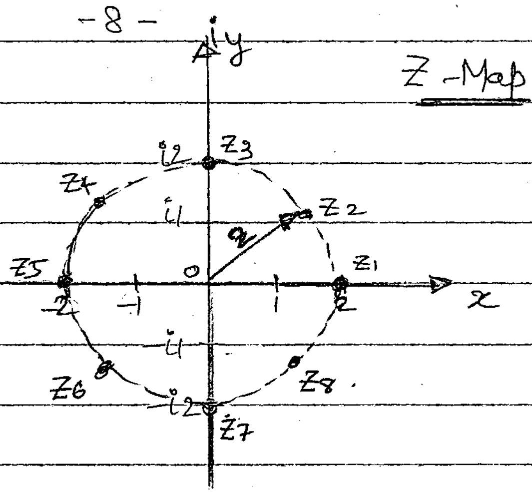

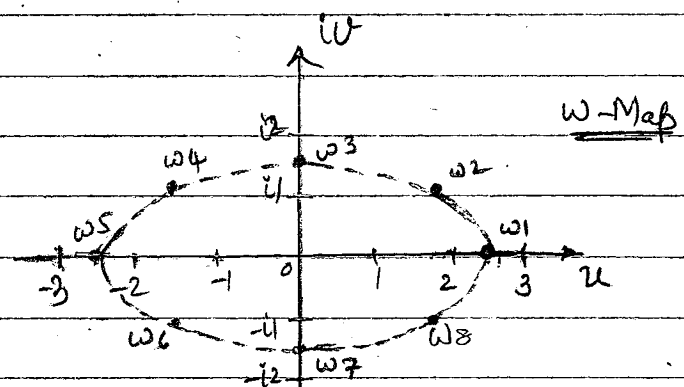

Hence the function $f(z) = z + \frac{1}{z}$ converts circle to an ellipse!!