U3A MATHS

10-Oct-2023 Term4 / Week1

Complex Function Plots

- During last few weeks, we plotted the complex function using (x, y) and (u, v) plots.

- The plots of complex function
$$\omega = f(z) = z + 1/z$$
where $z = (x+iy)$ &
$$\omega = (u+iv)$$
was particularly useful and was discovered by Joukowsky for aerofoil design

- We will now explore plotting of complex functions as a three dimensional plot.

- Note that a complex function requires 4 dimensions for plotting $(x, y, u \& v)$, but we live in a 3-dimensional world.
- Hence, we generally plot
  (1) x, y & Real part of z (u)
  (2) x, y & Imag part of z (v)
  (3) x, y & Mag. of z $(|z|) = \sqrt{u^2+v^2}$

The last one being more popular.

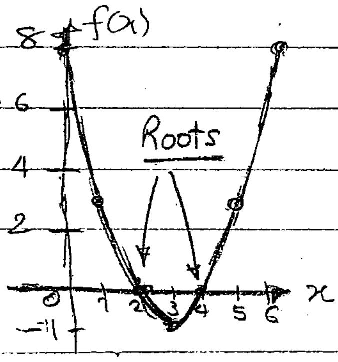
- Let us go back to the origin of complex numbers!

- Let us solve the quadratic equation:
$$f(x) = x^2 - 6x + 8$$

- We can calculate values of 'x' for $f(x)=0$ ie, Roots

We have $x = 2$ & 4

The plot of $f(x)$



The results are straight forward!

- Now let us solve:
$$f(x) = x^2 + 6x + 10 \Rightarrow ax^2+bx+c$$

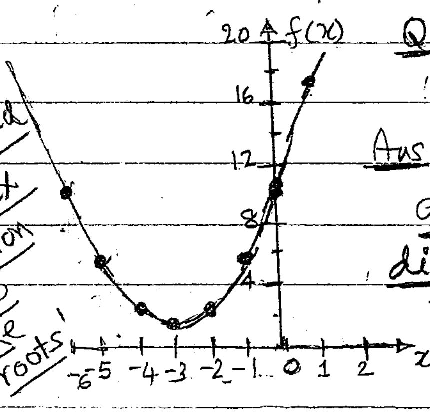
Solution is $\Rightarrow \dfrac{-b \pm \sqrt{b^2-4c}}{2a}$

$\therefore$ The roots are : $\dfrac{-3 \pm \sqrt{-4}}{2}$ or $(-3+i1)$ & $(-3-i1)$

Real Part -3 & Im Part ± 1

The plot of the function is

We need complex function plot to see the roots!



Q: Where are the Roots!

Ans: We need another dimension to see the Roots!

Complex Function Plots

The example quadratic is as given below:

f(x) = x^2 + 6x + 10 - Conventional equation using a 'real' variable.
- This cannot handle square root of a negative number!

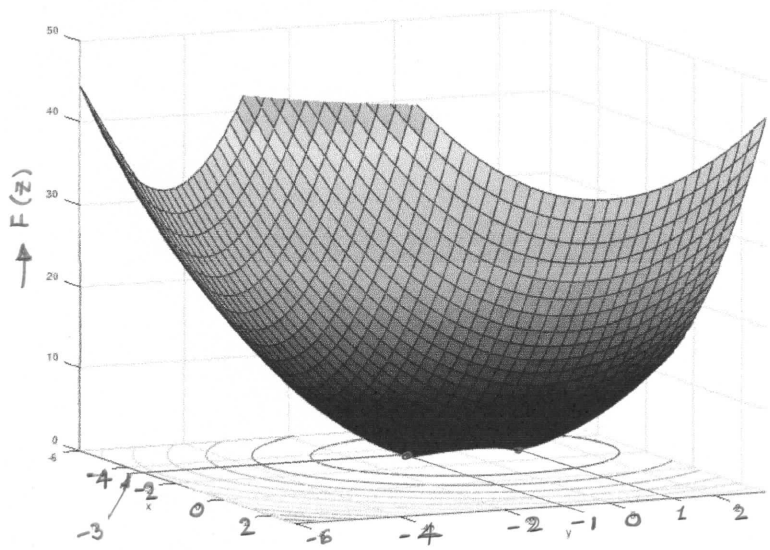
The same equation using complex variable is as given below:

F(z) = z^2 + 6z + 10 - Corresponding function in 'complex' form, where z = (x +iy)
- Note that the function values of F(z) is a complex number.
- This equation can handle square root of negative numbers!
- Note that F(z) = f(x) when 'y' is set to 0!

Octave program to plot F(z) and f(x)

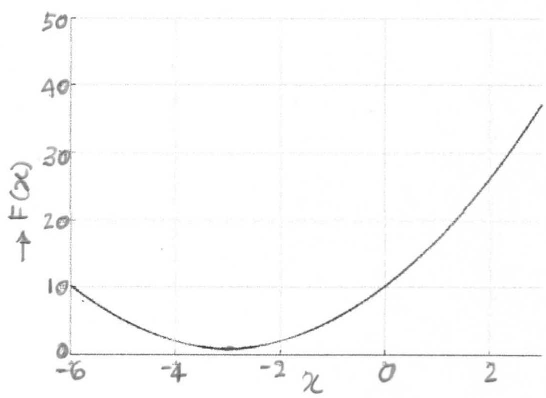
```
% Specify Input range
x = -6:0.25:3;
y = -6:0.25:3;
% y = -0.5:0.25:0.5;      # Provides input range for plotting f(x) = x^2 + 6x + 10

% Create meshgrid (data points) for complex (x-y) plane
% 'xx' matrix has 'x' values as rows, 'yy' has 'y' values as columns
[xx,yy] = meshgrid(x,y);

%  Claculate function values
z = xx + i*yy;              # Input data for complex plane
fz = z.*z + 6*z + 10;    # Plot for complex function f(z) = z^2 + 6z + 10

% Plot the function values
surfc(x,y,abs(fz));
axis([-6,3,-6,3,0,50]);
colormap jet;
xlabel('x');
ylabel('y');
```

Plot of F(z) = z^2 + 6z +10



x axis y axis

Plot of f(x) = x^2 + 6x + 10



Plot of f(x) corresponds to the plot of F(z) when y=0!

Root # 1 is at x = -3 & y = +1 and Root # 2 is at x = -3 & y = -1