U3A Maths

17-Oct-2023 Term 4 / Week 2

Complex Function Plots

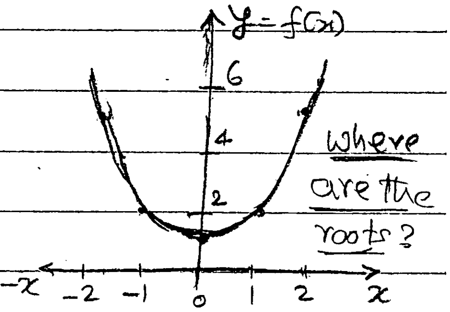
- Last week we plotted the complex function
  $f(z) = z^2 + 6z + 10$
- Let us now plot a much simpler complex function!
  $f(z) = z^2 + 1$ where $z = (x + iy)$
- If z has only "real" values then the function can be written as
  $f(x) = x^2 + 1$

- The above equation is simple to plot & to find the roots.

Roots are obtained by setting
$f(x) = 0$ or $x^2 + 1 = 0$
$\therefore x = \pm\sqrt{-1}$
("Imaginary" Roots!)

- The function plot is

x	$f(x)$
-2	5
-1	2
0	1
1	2
2	5



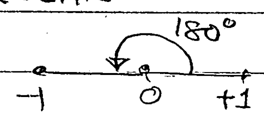
where are the roots?

- The roots are not seen on the above function plot, since we have an one-dimensional values for the variable 'x'.
- In other words, the "real" variable 'x' cannot be assigned a value of $\sqrt{-1}$, since our number system has no facility to represent "square root of negative numbers"

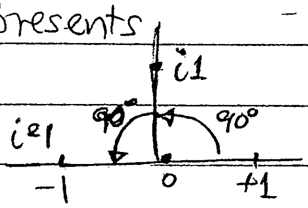
- let us represent $\sqrt{-1}$ by the symbol "i".

$\therefore$ We have, $i = \sqrt{-1}$ or $i^2 = -1$

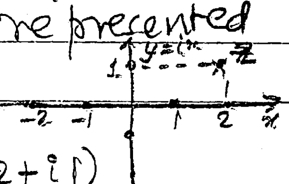
- Since $i^2 = -1$ represents a shift of $180°$,



hence $i = \sqrt{-1}$ represents a shift of $90°$



- Hence, a general purpose number can be represented as a combination of real & imag nos.



For Ex: $z = (2 + i1)$

- Hence, a general or complex number has two components or two dimensions!

$$z = x + iy$$

x — Real component; iy — "Imaginary" Component

- I prefer to use the term "quadrature" instead of "Imaginary".

- Let us now plot our complex form function, namely,

$$f(z) = z^2 + 1$$

where $z = (x + iy)$

Note: 'x' & 'y' values can be positive or negative numbers

- We need calculate the function values for given 'x' and 'y' values

Let $x = -2, -1, 0, 1, 2$ &

$y = -2, -1, 0, 1, 2$

- To simplify calculations, let us express the function in terms of the components, namely, x & y

$$z^2 + 1 = (x + iy)^2 + 1$$
$$= x^2 + i\,2xy - y^2 + 1$$
$$= \underbrace{(x^2 - y^2 + 1)}_{\text{Real Part}} + i\,\underbrace{(2xy)}_{\text{Imag Part}}$$

x	y	(x^2-y^2+1) (Real Part)	$(2xy)$ (Im. Part)	Abs. Val.
-2	-2	1	8	$\sqrt{1^2+8^2} = 8.062$
-1	-2	-2	4	4.472
0	-2	-3	0	3.0
1	-2	-2	-4	4.472
2	-2	1	-8	8.062
-2	-1	4	4	5.657
-1	-1	1	2	2.236
Root ⇒ 0	-1	0	0	0
1	-1	1	-2	2.236
2	-1	4	-4	5.656
-2	0	5	0	5.0
-1	0	2	0	2.0
0	0	1	0	1.0
1	0	2	0	2.0
2	0	5	0	5.0

Cont'd

x	y	x^2-y^2+1	$2xy$	Abs. Val
-2	1	4	-4	5.657
-1	1	1	-2	2.236
Root ⇒ 0	1	0	0	0
1	1	1	2	2.236
2	1	4	4	5.657
-2	2	1	-8	8.062
-1	2	-2	-4	4.472
0	2	-3	0	3.0
1	2	-2	4	4.472
2	2	1	8	8.062

Normally "Absolute Values" of functions are plotted.

Note: These values correspond to $f(x) = x^2 + 1$ since all 'y' values are '0'!

Complex Function Plots

The example quadratic is as given below:

f(x) = x^2 + 1
- Conventional equation using a 'real' variable.
- This cannot handle square root of a negative number!

The same equation using complex variable is as given below:

F(z) = z^2 + 1
- Corresponding function in 'complex' form, where z = (x +iy)
- Note that the function values of F(z) is a complex number.
- This equation can handle square root of negative numbers!
- Note that F(z) = f(x) when 'y' is set to 0!

Octave program to plot F(z) and f(x)

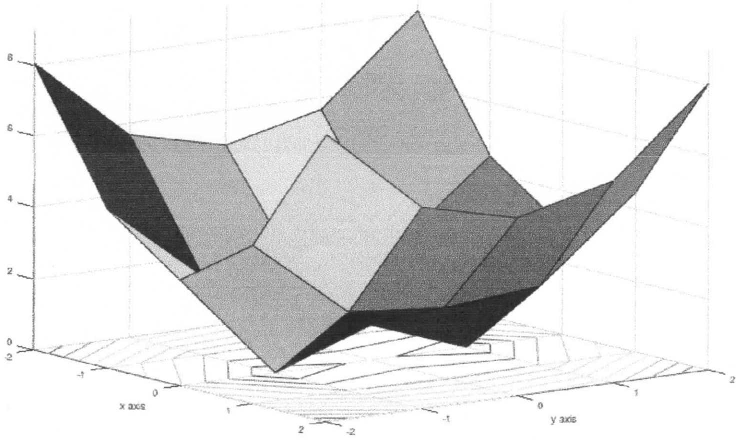
```
% Specify Input range
x = -2:1:2;
y = -2:1:2;
%y = -0.25:0.25:0.25;   # Provides plot for f(x) = x^2 + 1

% Create meshgrid (data points) for complex (x-y) plane
% 'xx' matrix has 'x' as rows, 'yy' has 'y' as columns
[xx,yy] = meshgrid(x,y);

%  Claculate function values
z = xx + i*yy;          # Input data for complex plane
fz = z.*z + 1;          # Plot for complex function f(z) = z^2 + 1

% Plot the function values
surfc(x,y,abs(fz));
axis([-2,2,-2,2,0,9]);
colormap jet;
xlabel('x axis');
ylabel('y axis')
```

Plot of F(z) = z^2 + 1



x axis y axis

Plot of f(x) = x^2 + 1