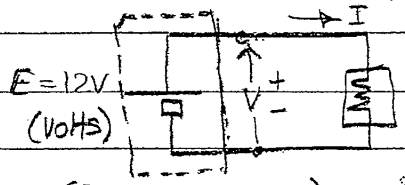
# U3A Maths

24-Oct-2023 Term 4 / Week 3

## AC Circuit Analysis

- Let us first consider DC networks to understand the circuit equations for analysis, which are applicable to AC circuits also.

Ex-1
- Consider a simple DC network






- Using Ohm's law

$$V = IR \quad \text{or} \quad I = \frac{V}{R}$$

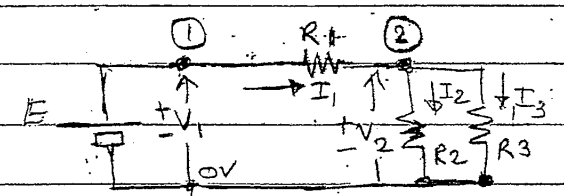
we have $V = E = 12V$ & $R = 12\Omega$

$$\therefore I = \frac{V}{R} = \frac{12V}{12\Omega} = 1A \text{ (Ampere)}$$

- Ohm's law
  - Georg Ohm (German: 1789-1854) experimented with current flow in various lengths of wire!
  - school teacher (with Ph.D!) taught maths & experimented in physics lab. wrote book on geometry to earn a decent income.



Ex.2 Let us now consider a more general network




Given: $E = 250V$, $R_1 = 10\Omega$

$R_2 = 100\Omega$, $R_3 = 400\Omega$

Find: Load voltage ($V_2$) & Total current ($I_1$)

- In general the above types of problems are solved by using ohm's at each node with unknown voltage.



- In addition, Kirchoff's law is used at each node
- At each node:

Sum of currents flowing IN = Sum of currents flowing OUT

In general, we can write

$$\sum I_{IN} = \sum I_{out}$$

∴ At node (2), we have

$$I_1 = I_2 + I_3$$

Now apply ohm's law

$$\frac{(V_1 - V_2)}{R_1} = \frac{(V_2 - 0)}{R_2} + \frac{(V_2 - 0)}{R_3}$$

Note: current directions must be assigned, but can be arbitrary!

$$\frac{(250-V_2)}{1\Omega} = \frac{V_2}{10\Omega} + \frac{V_2}{40\Omega}$$

$$250 - V_2 = 0.1V_2 + 0.025V_2$$

$$250 = V_2 + 0.1V_2 + 0.025V_2$$

$$\therefore V_2 = \frac{250}{1.125} = 222.22$$

$$\therefore I_1 = \frac{E - V_1}{R_1} = \frac{250 - 222.22}{10}$$

Ohm's law again!

$$= 2.178\ A$$

The above is a practical problem — used for sizing the cable so that load Voltage ($V_2$) is acceptable!

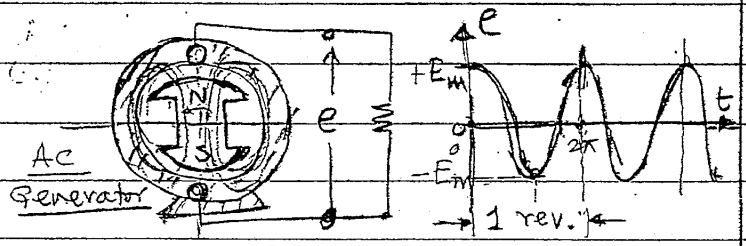
- Gustav Kirchoff (German, 1824-87) - Electric circuits, spectroscopy, Black body radiation



## A.C. Systems

- In D.C. systems, the generator voltage and hence the current remain constant with respect to time
- In A.C. systems, the generator voltage and hence the current varies sinusoidally w.r.t. time.
- Sinusoidal variation is obtained by appropriate design of the A.C. generator.
- Hence, the first requirement is to represent the A.C. voltage & current mathematically.






- The number of cycles/sec depends on no. of revolutions per second of the rotor.
- For our 2-pole generator

1 revolution = 1 cycle

∴ 50 rev/sec = 50 cycles/sec

(3000 rpm) (rev./min) (say "f") (frequency)

- We need an equation to find the voltage value at any given time, say "t" secs.



1 cycle = $2\pi$ radians

1 sec = $2\pi \times f$ radians

∴ t sec $\Rightarrow 2\pi ft$ radians

∴ we can write that the voltage at any given time 't' as below:

$$e(t) = E_m \cdot \cos(2\pi ft) \text{ Volts}$$

Ex: Given that $E_m = 250V$ & $f = 50$ find the voltage at $t = 0.01$ sec.

$$e(t) = E_m \cos(2\pi ft)$$

$$= 250 \cos(2\pi \times 50 \times 0.01)$$

$$= -250\ V\ !$$