U3A Maths

31-Oct-2023 Term 4 / Week 4

A.C. Circuit Analysis

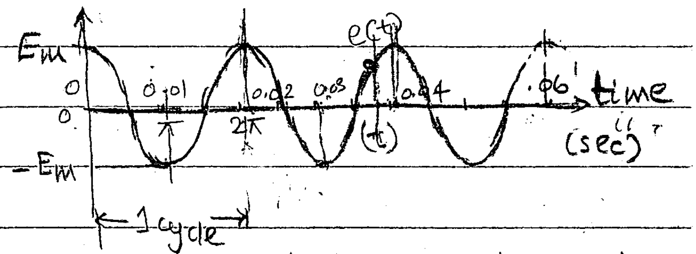
- Review
  - Ohms Law $I = V/R$
  - Kirchoff's Current Law (KCL) $\sum I_{in} = \sum I_{out}$
  - General Circuit Analysis
    - Use KCL at each node with unknown voltage.
    - Express brach currents using Ohm's Law.
  - The above is applicable for both DC & AC circuit analysis.

-2-

A.C. Systems

- In AC systems, the generator voltage (as per design) varies sinusoidally w.r.t. time
- Hence, current also varies sinusoidally w.r.t. time.

- We represent the voltage variation w.r.t. time as below:



1 cycle ⇒ 1 revolution of a two-pole generator.

-3-

1 revolution = 1 cycle

∴ 50 rev/sec ⇒ 50 cycles/sec
(3000 rpm)

In general, say, "f" cycles/sec

(where 'f' is called the frequency)

- We need an equation to find the value of the voltage at any given time, say, "t" seconds
- We know that the variation is sinusoidal, and sinusoidal values are calculated using 'sine' or 'cosine' function for a given angle in 'degrees' or 'radians'

-4-

1 cycle = 2π radians

Also, 1 second ⇒ 'f' cycles

∴ t sec ⇒ $t \times f \times 2\pi$ radians

∴ We can represent the generator voltage at any given time 't'

$$e(t) = E_m \cdot \cos(2\pi f t) \text{ Volts}$$

(for the cosine wave shown in the figure.)

Ex: Given that $E_m = 250V$ and frequency $f = 50$ cycles/sec, Find voltage at

(a) 0.01 s (b) 0.038 s (c) 1.54 sec

we have:

$$e(t) = E_m \cos(2\pi f t) \text{ volts}$$
$$= 250 \cos(2\pi \times 50 \times t)$$

(a) $t = 0.01\,s$

$$e(0.01) = 250 \cos(2\pi \times 50 \times 0.01)$$
$$= 250 \cos(\pi)$$
$$= -250\,V$$

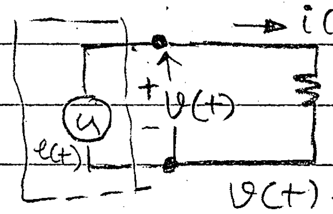
(b) $t = 0.22\,s$

$$e(0.22) = 250 \cos(2\pi \times 50 \times 0.038)$$
$$= 250 \cos(11.938)$$
$$= 202.25\,V$$

(c) $t = 1.54\,s$

$$e(1.54) = 250 \cos(2\pi \times 50 \times 1.54)$$
$$= 250 \cos(483.805)$$
$$\simeq 250\,V$$

Ex: Calculate the current flowing in the AC circuit below: Given $e(t) = 250 \cos(2\pi \times 50 \times t)$ Volts.

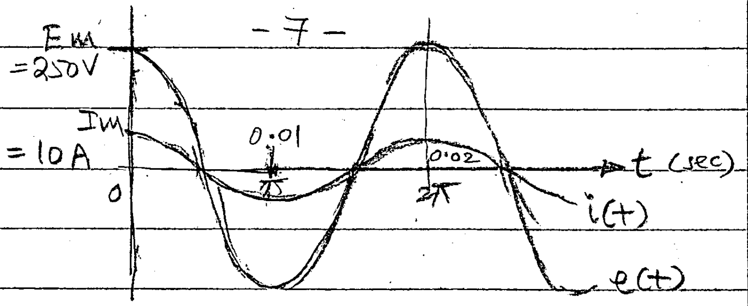


$v(t) = e(t)$

(Note: voltage polarity is consistent with current direction. They both change direction every half cycle!)

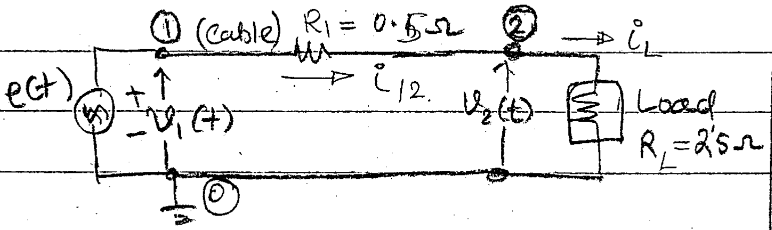
Using Ohm's law:

$$i(t) = \frac{v(t)}{R} = \frac{250 \cos(2\pi \times 50 \times t)}{25}$$
$$= 10 \cos(2\pi \times 50 \times t)$$



Let us consider another example

Ex: Find the current in the AC circuit below:



Given $v_1(t) = e(t) = 240 \cos(2\pi \times 50 \times t)$ Volts

Using KCL: $i_{12} = i_L$

$$\frac{v_1(t) - v_2(t)}{0.5} = \frac{v_2(t)}{25}$$

Solving we get:

Load Volts: $v_2(t) = 235.3 \cos(2\pi \times 50 \times t)$ Volts

Load Current: $i_L(t) = 9.41 \cos(2\pi \times 50 \times t)$ Amps.

However, in A.C. systems, the load voltage $V_2(t)$ will be much lower due to the voltage drop caused by varying mag. flux! (No such issue in DC!!)