U3A Maths

7-Nov-2023 Term 4/Week5

A.C. Circuits - Inductance

- Solution of A.C. circuits becomes more complex (!) due to variation of magnetic field (flux) due voltage and hence current variations.

- The magnetic field (flux)(ϕ) is proportional to current (i) ie,

$$\phi \propto i$$

If 'i' varies sinusoidally then flux (ϕ) also varies sinusoidally

- The constant of proportionality can be established and is constant for a given magnetic device (circuit)

- Hence, we can write

$$\phi = L\, i$$

where L is called the "Inductance" of the given magnetic circuit.

- As per Faraday's law voltage induced due to varying magnetic field is

$$v_L = \frac{d\phi}{dt}$$

$\therefore$ we have,

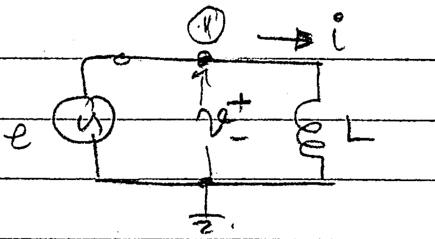
$$v_L = \frac{d\phi}{dt} = L \cdot \frac{di}{dt}$$

$$\therefore \quad v_L = L \cdot \frac{di}{dt}.$$

- Hence, the solution of A.C. circuits involves solution of differential equations!

- In general, we need to solve multiple (simultaneous) differential equations.

- Let us first consider a single inductance circuit.



Solve for 'i' for given

$$e = E_m \cos(2\pi f t)$$

For simplicity let $\omega = 2\pi f$

$$\therefore \quad e = E_m \cos(\omega t)$$

We have

$$e = v = L \cdot \frac{di}{dt}$$

$$\therefore \quad i = \frac{1}{L} \int e\, dt$$

$$= \frac{1}{L} \int E_m \cos(\omega t)\, dt$$

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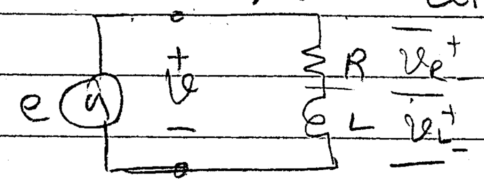
- Even though the above integration is straight forward, but in general it is not simple to solve Ac networks.

For Ex; for the following circuit



$$e = v = v_R + v_L$$

$$E_m \cos(\omega t) = iR + L.\frac{di}{dt}$$

-Solution is not straight forward !!

- There is a much smarter way to solve the above problem - using Complex Numbers!

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- Considering our Inductance equation again

$$i = \frac{1}{L}\int E_m \cos(\omega t)\, dt$$

Let us express $\cos(\omega t)$ as

$$\cos(\omega t) = \text{Real Part of } \left[e^{j\omega t}\right]$$

$$= Re...\left[\cos\omega t + j\sin\omega t\right]!$$

Note that we have used $j = \sqrt{-1}$ instead of 'i' since we have used 'i' to represent current.

$$\therefore\ i = \frac{E_m}{L}\int e^{j\omega t}\, dt$$

[We can use the "real part" later]

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hence, $i = \dfrac{E_m}{(j\omega)L} \cdot e^{j\omega t}$

$$= \frac{E_m}{(j\omega L)} e^{j\omega t}$$

$$= I_m\, e^{j\omega t}$$

$$I_m \cancel{e^{j\omega t}} = \frac{E_m}{j\omega L} \cdot \cancel{e^{j\omega t}}$$

$$\therefore\ \boxed{I_m = \frac{E_m}{j\omega L}}$$

In practice, we are only interested in the "peak" value, since we know all Ac waves are sinusoidal.

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In fact, for Resistance we can write:

$$i = \text{Real Part of} \left[\frac{E_m e^{j\omega t}}{R}\right]$$

$$= I_m \cdot e^{j\omega t}$$

where $\boxed{I_m = \dfrac{E_m}{R}}$

We now have a general purpose Ohm's law for Ac circuits

For Resistance (R) $\Rightarrow I_m = \dfrac{E_m}{Z}$ where $Z = (R + j0)$

& for Inductance (L) $\Rightarrow I_m = \dfrac{E_m}{Z}$ where $Z = (0 + j\omega L)$

We can now solve for the load voltage (v_2) for the Ex. in Week 4!