U3A Maths

14-Nov-2023 Term 4/Week 6

A.C. Circuit Solution

- Solution of A.C. Circuits becomes complicated, due to variation of magnetic field (flux) and also variation of electric field (flux) associated A.C. currents & voltages.

- The effect of variation of magnetic field (flux) is modelled (mathematically) by an "Inductance"

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- We have; voltage drop due to inductance:

$$v_L = \frac{d\phi}{dt} = L\frac{di}{dt}$$

or $$i = \frac{1}{L}\int v_L\,dt$$

- Similarly, effect of electric field is modelled by a Capacitance.

- We have charge (q) due to electric field due to the voltage (v_c)

$$q = C \cdot v_c$$

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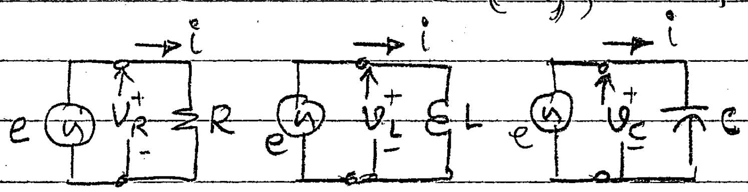
Hence, the current flow

$$i = \frac{dq}{dt} = C \cdot \frac{dv_c}{dt}$$

- Given $e = E_m \cos(\omega t)$, where $\omega = 2\pi f$



$$\underbrace{i = \frac{E_m \cos(\omega t)}{R}} \quad \underbrace{i = \frac{E_m}{L}\int \cos(\omega t)\,dt} \quad \underbrace{i = E_m C \cdot \frac{d(\cos\omega t)}{dt}}$$

- The calculations become much simpler, if we replace "$E_m \cos(\omega t)$" by

$$\cos(\omega t) = \text{Real part of } \left[E_m e^{j\omega t}\right]$$

where $j = \sqrt{-1}$

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- In other words, we replace $E_m \cos(\omega t)$ by the corresponding complex number in Euler's form.

- We can keep "Real part of" at the back of our mind and write

For Resistance

$$i = \left(\frac{E_m}{R}\right) e^{j\omega t}$$

For Inductance

$$i = \frac{E_m}{L}\int e^{j\omega t} = \frac{E_m}{j\omega L} \cdot e^{j\omega t}$$

For Capacitance

$$i = E_m C \frac{d}{dt}(e^{j\omega t}) = E_m (j\omega C) e^{j\omega t}$$

- If we are interested only in magnitudes of voltages & currents, we can ignore $e^{j\omega t}$ as it essentially represents sinusoidal variation.
- Hence, it is common to ignore $e^{j\omega t}$ in practice!

∴ we have ;-

For Resistance

$$I_m = \frac{E_m}{R}$$

For Inductance

$$I_m = \frac{E_m}{j\omega L}$$

For Capacitance

$$I_m = \frac{E_m}{(1/j\omega C)}$$

Hence, we can now write generalised "Ohm's Law" for A.C.

$$I_m = \frac{E_m}{Z}$$

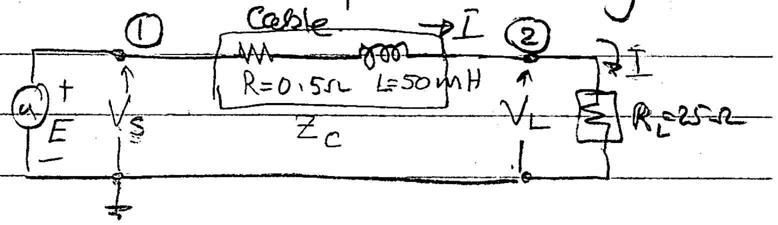
where $Z = (R + j0)$ – Resistance

$= (0 + j\omega L)$ – Inductance

$= \left(0 + \frac{1}{j\omega C}\right)$ – Capacitance

where Z is called "Impedance"

Ex: Let us now solve the problem in Week 4, by considering inductive drop due to mag. field



Find load voltage V_L, given that $E = V_S = 240\,V$ & $f = 50\,Hz$

(Note: we are only considering the magnitudes (peak values?) of voltages & currents)

We have:

$$\omega = 2\pi f = 2\pi \times 50 = 314.16$$

∴ Cable impedance

$$Z_C = R + j\omega L$$
$$= 0.5 + j\,314.16 \times 50 \times 10^{-3}$$
$$= (0.5 + j15.708)\ \Omega$$

Also Load Impedance:

$$Z_L = (R_L + j0)$$
$$= (25 + j0)$$

Using KCL at node 2

$$\frac{(V_S - V_L)}{Z_C} = \frac{V_L}{Z_L}$$

$$\frac{(240 - V_L)}{(0.5 + j15.708)} = \frac{V_L}{(25 + j0)}$$

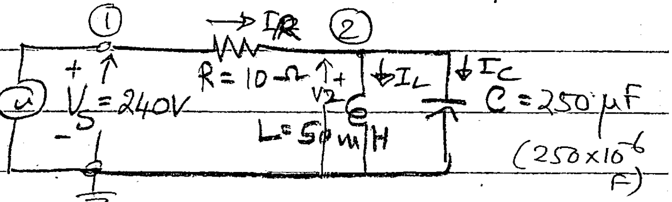
Solving, we get

$$V_L = (170.57 - j\,105.07)\ V$$

∴ Magnitude of $V_L = |V_L| \approx \underline{200\,V}$

Motor will not start!

Home Work Calculate the voltage at Node 2 (V_2) & currents I_R, I_L & I_C



Ans

$V_2 = (234.803 - j34.934)$ $I_R = (0.52 - j3.493)$

$|V_2| = \underline{237.79\,V}$ $|I_R| = \underline{3.532\,A}$

$I_L = (-2.224 - j\,14.948)$ $I_C = (2.744 + j\,18.441)$

$|I_L| = \underline{15.113\,A}$ $|I_C| = \underline{18.644\,A}$