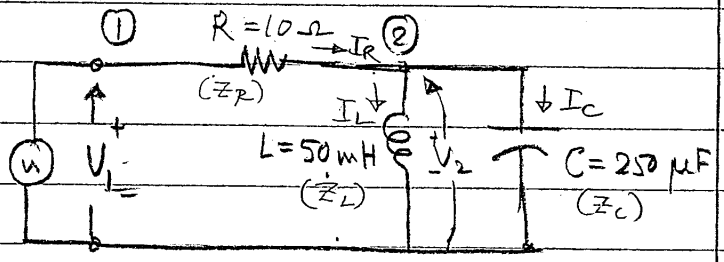
U3A Maths

14-Nov-2023 Term 4 / Week 6

A.C. Circuits - Homework

Given $V_1 = 240V$ & $f = 50Hz$

Solve the following circuit for node voltage (V_2) and branch currents I_R, I_L & I_C.



$\omega = 2\pi f = 2\pi \times 50$

$= 314.16$ radians

Let us first calculate the impedances in complex form:

Resistance: $Z_R = (R + j0) = (10 + j0)\ \Omega$

Inductance: $Z_L = (0 + j\omega L)$

$= (0 + j\ 314.16 \times 50 \times 10^{-3})$

$= (0 + j\ 15.708)\ \Omega$

Capacitance: $Z_C = (0 + \frac{1}{j\omega C})$

$= \left(0 + \frac{1}{j\ 314.16 \times 250 \times 10^{-6}}\right)$

$= (0 - j\ 12.732)\ \Omega$

Using KCL at node ②

$I_R = I_L + I_C$

i.e., $\frac{(V_1 - V_2)}{Z_R} = \frac{V_2}{Z_L} + \frac{V_2}{Z_C}$

$$\frac{(240 - V_2)}{(10 + j0)} = \frac{V_2}{(0 + j15.708)} + \frac{V_2}{(0 - j12.732)}$$

$$\frac{240}{10} = V_2\left[\frac{1}{10} + \frac{1}{j15.708} + \frac{1}{j12.732}\right]$$

i.e., $24 = V_2\,[0.1 - j0.0637 + j0.0785]$

$$\therefore V_2 = \frac{24}{(0.1 + j0.014878)}$$

$$= (234.803 - j34.934)$$

$\therefore |V_2| = V_2$ Magnitude $= 237.39V$

Note: Final values are as per exact calculations using Octave!

Currents are:

$$I_R = \frac{V_1 - V_2}{Z_R} = \frac{(240 - (234.803 - j34.934))}{(10 + j0)}$$

$$= (0.52 + j3.493)$$

$\therefore |I_R| = 3.532\ A$

$$I_L = \frac{V_2}{Z_L} = \frac{(234.803 - j34.934)}{(0 + j15.708)}$$

$$= (-2.224 - j14.948)$$

$\therefore |I_L| = 15.113\ A$

$I_C = \frac{V_2}{Z_C}$ i.e. $I_C = (2.743 + j18.441)$

$\therefore |I_C| = 18.644A$

- currents in L & C branches are higher than the current in R!
- This feature is used in Radio transmission!
- $I_R = I_L + I_C$ is still True in complex form!!