U3A Maths

13-Feb-2024 Term 1 / Week 3

Binary Numbers

- In decimal system, we have 10 symbols — namely 0 to 9
- A positional system with a weightage of powers of 10 are used for larger numbers: For Ex.

$12 = 1\times10^1 + 2\times10^0 = 12$

$432 = 4\times10^2 + 3\times10^1 + 2\times10^0 = 432$

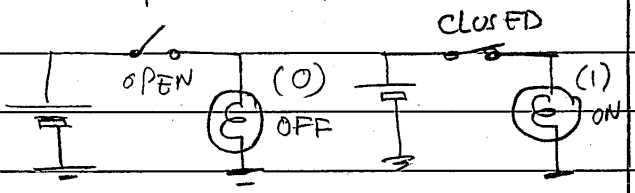
(10^2 10^1 10^0)

- The above logic can be used to develop number systems with more or

less than 10 symbols.

- We are now interested in a number system with 2 symbols, namely 0 and 1
- Such a system is called a Binary Number System
- Binary numbers are used in the design of computers, as they provide for reliable design of computers using electrical circuits.

- In its simplest form, binary numbers can be implemented by a simple switch.

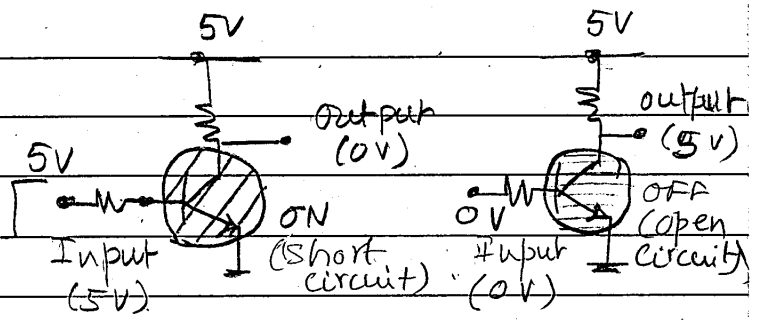


Switch open ⇒ Lamp OFF ⇒ 0

Switch closed ⇒ Lamp ON ⇒ 1

- Note that there is no ambiguity in this design, the switch can only be 'open' or 'closed'

- Electronically this is implented using a Switching transistors



- The power of the digital computer is due to the fact that, switching can be done at a speed of a microsecond (10^{-6}s) to a few nanoseconds (10^{-9})

- Binary Numbers

Binary	Decimal Equivalent
0	0
1	1
10	2
11	3
100	4
101	5
110	6
111	7

- In general, to convert a Binary to decimal number:

Ex: $10111 = 1\times2^4 + 0\times2^3 + 1\times2^2 + 1\times2^1 + 1\times2^0$

$2^4\ 2^3\ 2^2\ 2^1\ 2^0$

$= 16 + 0 + 4 + 2 + 1\times1$

$= 23_{(10)}$

- Converting a decimal number into Binary

Ex: 23

		Remainder
2	23	
2	11	1
2	5	1
2	2	1
	1	0 ↑

∴ Binary number is

$(23)_{10} = 10111$

- A binary decimal (!) [fraction] can be written as

10.11

$2^1\ 2^0 . 2^{-1}\ 2^{-2}$

$= 1\times2^1 + 0\times2^0 . 1\times2^{-1} + 1\times2^{-2}$

$= (2+0) . (0.5 + 0.25)$

$= 2.75$

- Fractions in digital computers are represented in a different way, - which we will cover at a later date.

- The main disadvantage of the binary system is that is that it needs more digits for a given number in decimal.

- Typically, about 2 to 3 times the number of digits in decimal

For Ex: $7 \Rightarrow 111$ (1:3)

$15 \Rightarrow 1111$ (1:2)

$31 \Rightarrow 11111$ (1:2.5)

$63 \Rightarrow 111111$ (1:3)

- Hence, about 8 digit accuracy can be obtained with 24 binary digits!