U3A Maths

27-Feb-2024 Term 1 / Week 5

Subtraction - Complement Addition

Home work

$$\begin{array}{r} 4285 \\ -\ 928 \\ \hline +3357 \end{array} \quad \xrightarrow{\text{9's compl.}} \quad \begin{array}{r} 4285 \\ +\ 9071 \\ \hline \text{Carry } 1\ 3356 \\ 1 \\ \hline +3357 \end{array}$$

$$\begin{array}{r} 0526 \\ -\ 4829 \\ \hline -\ 4303 \end{array} \quad \xrightarrow{\text{9's compl.}} \quad \begin{array}{r} 526 \\ 5170 \\ \hline 5696 \end{array}$$

No carry!

$5696 \xrightarrow[\text{9's comp.}]{} -4303$

It is a negative no. ∴ Take complement.

History of Complement!

- Complement method was widely used during 1700's & 1800's
- Thomas Dilworth wrote a book in 1802 advocating the use of complement!
- In fact, Blaise Pascal (French) designed and built a mechanical Calculator in 1642 (aged 18). His machine used complement method for subtraction - since the "adder" could perform subtraction!

- Pascal was a mathematician, physicist, inventor, philosopher and a Catholic writer!
- Pascal's Gamble:
  - * It is better to believe in God *
  - If God does not exist, then an individual incurs some losses, by sacrificing some pleasures & luxuries
  - If the God does exist, then you gain much more & you are also in big trouble for ignoring God!

∴ On the balance, it is safer to believe in God!

Complement in Binary System

- Instead of 9, we use 1's complement

$0 \xrightarrow{\text{1's complement}} 1 - 0 = 1$

$1 \xrightarrow{\text{1's complement}} 1 - 1 = 0$

Number	1's complement
0 ⇒	1
1 ⇒	0

Let us now do subtraction

Ex:

$$\begin{array}{r} 12 \\ -\ 7 \\ \hline 5 \end{array} \qquad \begin{array}{r} 1100 \\ -\ 0111 \\ \hline \end{array} \quad \xrightarrow{\text{1's compl}} \quad \begin{array}{r} 1100 \\ +\ 1000 \\ \hline 1\ 0100 \\ \text{Carry} \rightarrow 1 \\ \hline 0101 \\ (=5) \end{array}$$

Ex

5	0101	0101
−12	− 1100	+ 0011
−7		1000

No carry

∴ Negative No.

$1000 \xrightarrow[\text{Compl.}]{1's} -0111\ (-7)$

Note

- There is also 2's complement method which has some advantages, but we will stick to 1's complement.

Representation of Negative Numbers in Computers

- In computers, it is convenient to use the most significant binary digit (bit) to represent the sign rather than using '+' and '−' symbols.

- For example, considering a 4-bit machine,

$\underset{\text{sign}'+'}{0}\underbrace{101}_{\text{Value}} \Rightarrow +5$

and $\underset{\text{Sign}'-'}{1}\underbrace{101}_{\text{Value}} \Rightarrow -5$

- However, it is more convenient to store the negative number in 1's complement form instead "signed" numbers.

- For example −5 is stored as 1's complement of +5

$+5 \Rightarrow 0101$

$-5 \Rightarrow 1010$ (1's compl.)

- Note that most significant bit still acts as the sign bit!

- Range of a 4 bit machine.

Sign bit (+)

0000	⇒ +0	
0001	⇒ +1	
0010	⇒ +2	
0011	⇒ +3	
0100	⇒ +4	
0101	⇒ +5	
0110	⇒ +6	
0111	⇒ +7	

Sign bit (−)

1000	$\xrightarrow[\text{Comp.}]{1's}$	0111	⇒ −7
1001	⇒	0110	⇒ −6
1010	⇒	0101	⇒ −5
1011	⇒	0100	⇒ −4
1100	⇒	0011	⇒ −3
1101	⇒	0010	⇒ −2
1111	⇒	0000	⇒ −0 !

- +0 is different from −0 ! (2's complement solves this problem)