U3A Maths

05-Mar-2024 Term 1 / Week 6

Binary Fractions

Review

- We can use 1's complement for subtraction and for storing negative numbers.
- This helps to perform subtraction using only the "Adder". We don't need to design a "Subtractor"!
- 1's complement example:

$+5 \Rightarrow 0101$

$-5 \Rightarrow 1010$ (1's complement)

- In computers, the most significant digit is used to indicate the "sign"

  0 ⇒ Positive

  1 ⇒ Negative

- The 1's complement system automatically provides this feature
- Considering a 4 bit machine

Sign bit (+)

0000 ⇒ +0	1000 $\xRightarrow{1\text{'s comp}}$ 0111 ⇒ −7
0001 ⇒ +1	1001 ⇒ 0110 ⇒ −6
0010 ⇒ +2	1010 ⇒ 0101 ⇒ −5
0011 ⇒ +3	1011 ⇒ 0100 ⇒ −4
0100 ⇒ +4	1100 ⇒ 0011 ⇒ −3
0101 ⇒ +5	1101 ⇒ 0010 ⇒ −2
0110 ⇒ +6	1110 ⇒ 0001 ⇒ −1
0111 ⇒ +7	1111 ⇒ 0000 ⇒ −0

- Note that we have separate codes for +0 (0000) & −0 (1111)
- This can be avoided by using 2's complement. But, 1's complement is adequate for our conceptual purposes.

Binary Fractions

- Fractional part is generally expressed as digits following a "(decimal!) point"!!
- So, we are often stuck with the terminology "Decimal" even for binary fractions!

For Ex: $\overset{10^0}{4}\,.\,\overset{10^{-1}}{5}\,\overset{10^{-2}}{2}\,\overset{10^{-3}}{5}$

$= 4\times10^0 + 5\times10^{-1} + 2\times10^{-2} + 5\times10^{-3}$

$= 4 + 0.5 + 0.02 + 0.005$

$= 4.525$

- In binary system we have a base of '2', hence, for Ex: $\overset{2^0}{1}\,.\,\overset{2^{-1}}{1}\,\overset{2^{-2}}{0}\,\overset{2^{-3}}{1}$

We can convert to decimal equilent as below:

$= 1\times2^0 + 1\times2^{-1} + 0\times2^{-2} + 1\times2^{-3}$

$= 1\times1 + 1\times\frac{1}{2^1} + 0\times\frac{1}{2^2} + 1\times\frac{1}{2^3}$

$= 1 + 1\times\frac{1}{2} + 0\times\frac{1}{4} + 1\times\frac{1}{8}$

$= 1 + 0.5 + 0 + 0.125 = (1.625)_{10}$

-5-

- Just as an exercise, let us convert a decimal fraction into binary fraction.

Ex.1 Convert 0.625 to binary

0.625×2

$1 \leftarrow 1.250$

0.25×2

$0 \leftarrow 0.50$

0.5×2

$1 \leftarrow 1.0$

Starting from top, the binary value is $(0.101)_2$

-6-

Note
- For conversion we need to treat fractional part separately

Ex:2 0.35×2

$0 \leftarrow 0.70 \times 2$

$1 \leftarrow 1.40 \times 2$

$0 \leftarrow 0.80 \times 2$

$1 \leftarrow 1.60 \times 2$

$1 \leftarrow 1.20 \times 2$

$0 \leftarrow 0.40 \times 2$

Endless repeat!

The binary value is

$(0.010110.....)_2$

We will need to round it off!

-7-

- The computers store and operate numeric values in two different modes (methods).

(1) Integer mode (whole numbers)

(2) Floating Point mode (numbers with associated fraction)

- For most arithmetic calculations, floating point mode is used.

-8-

- What is floating point?

- It is a method of representing a fractional number in a standardised format.

Ex:1 25.16 — Given no.

0.2516×10^{2} ⇒ Floating Point

Ex.2 $0.025 \Rightarrow 0.25 \times 10^{-1}$

Ex.3 $1024 \Rightarrow 0.1024 \times 10^{4}$