U3A Maths

12-Mar-2024 Term 1 / Week 7

2's Complement

- 2's complement is defined as 1's complement plus 1.

- Examples:

(a) Find 2's complement of 1011

$1011 \xrightarrow[\text{Complement}]{1's} 0100$

$0100 + 1 = 0101$

2's complement $\Rightarrow 0101$

(b) Find 2's complement of 0111

$0111 \xrightarrow[\text{Complement}]{1's} 1000$

$1000 + 1 = 1001$

2's complement 1001

(c) Find 2's complement of 1001

$1001 \xrightarrow[\text{complement}]{1's} 0110$

$0110 + 1 = 0111$

Note: we get back the given number in (b) above.

- 2's complement is reversible using the same process.

- Subtraction Using 2's Complement

(a) $12 \Rightarrow 1100$

$-4 \Rightarrow -0100$

8

2's complement of $0100 \Rightarrow 1011$

$1011 + 1 = 1100$

$\therefore$ $1100 + 1100$ ← Add complement

$11000 \Rightarrow 8$

Ignore carry

(b) $4 \Rightarrow 0100$

$-6 \Rightarrow -0110$

-2

2's complement of $0110 \Rightarrow 1001$

$1001 + 1 = 1010$

$\therefore$ $0100 + 1010$ ← Add complement

01110

No carry! $\therefore$ it is a negative number.

$\therefore$ Take 2's complement to get the value!

$1110 \xrightarrow[\text{Compl.}]{2's} 0001$

$0001 + 1 = 0010 = (2)_{10}$

- The main advantage of 2's complement is that, there is no need to add carry during subtraction. Hence, it leads to a more efficient design. Of course, 1 needs to be added to obtain 2's complement, which negates the above advantage.
- However, 2's complement has another advantage while storing negative numbers in complement form!

- Let us first consider 1's complement and a 4-bit machine, with the "left most digit" indicating the sign; 0 is +ive & 1 is -ive.
- Range of 1's complement

		1's complement	
0 000	+0	1 111	-0
0 001	+1	1 110	-1
0 010	+2	1 101	-2
0 011	+3	1 100	-3
0 100	+4	1 011	-4
0 101	+5	1 010	-5
0 110	+6	1 001	-6
0 111	+7	1 000	-7

- The range is +7 to -7 and we have +0 & -0, which could be confusing; -0 is complement of +0!
- Let's check 2's complement:

		2's compl.	
0 000	+0	1 0000 (ignore carry)	+0
0 001	+1	1 111	-1
0 010	+2	1 110	-2
0 011	+3	1101	-3
0 100	+4	1100	-4
0 101	+5	1011	-5
0 110	+6	1010	-6
0 111	+7	1001	-7
		1000 ??	(-8)

1111 + 1 = 0000

This "Extra" bit pattern is available

- The "Extra" bit pattern is a negative number, since left most digit is '1'.
- So, we need to find 2's complement to establish the value.

1000 —(2's Compl.)→ 0111 + 1 = 1000

- The converted number is a positive number, hence the value -8!
- Hence, the range is +7 to -8 and there is only +0!
- Due to these advantages, 2's complement is more popular!