

Binary Fractions

Review

A negative binary number can be represented as follows:

- (1) Left most bit used as sign.
0 \Rightarrow +ive 1 \Rightarrow -ive
- (2) 1's complement form
Left most bit indicates the sign! 0 \Rightarrow +ive 1 \Rightarrow -ive
- (3) 2's complement form
Left most bit indicates the sign! 0 \Rightarrow +ive 1 \Rightarrow -ive

options (1) & (2) result in different bit patterns for +0 & -0 [Considering 4 bit m/c]

- Ex (1) signed number 0000 & 1000
(2) 1's complement 0000 & 1111

The 2's complement has only +0. Hence, 2's complement form is used as the standard form in modern digital computers for negative numbers.

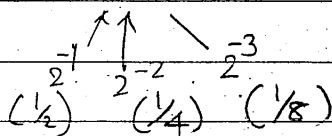
As a bonus, the extra bit pattern is used to represent additional negative number. For 4 bit m/c 1000 $\xrightarrow{2's\ complement}$ +1000 (-8)!

Binary Fractions

Fractional part in binary is usually expressed using a binary point! (Analogous to decimal point)

For example, consider

0.101



Note: $2^{-1} = \frac{1}{2} = 0.5$
 $2^{-2} = \frac{1}{2^2} = \frac{1}{4} = 0.25$
 $2^{-3} = \frac{1}{2^3} = \frac{1}{8} = 0.125$

Hence, the value in decimal form is

$$= 1 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3}$$

$$= 1 \times 0.5 + 0 \times 0.25 + 1 \times 0.125$$

$$= \underline{0.625}$$

$\therefore (0.101)_2 = (0.625)_{10}$

Ex. 2 Convert

11.11 to decimal

Convert whole and fractional parts separately.

$$\begin{matrix} 2^1 & 2^0 & 2^{-1} & 2^{-2} \\ (1 & 1 & 1 & 1)_2 \end{matrix}$$

$$= 1 \times 2^1 + 1 \times 2^0 + \frac{1 \times 2^{-1}}{0.5} + \frac{1 \times 2^{-2}}{0.25}$$

$$= (3.75)_{10}$$

Just as an exercise, let us convert a decimal fraction into binary fraction

Ex: Convert $(0.625)_{10}$ to binary

$$\begin{array}{l} 0.625 \times 2 \\ \downarrow 1 \leftarrow \text{starting} \\ 0.25 \times 2 \quad \text{from top-} \\ 0 \leftarrow \text{the value is} \\ 1 \leftarrow 0 \quad (0.101)_2 = (0.625)_{10} \end{array}$$

Floating Point Number

In computers, numbers with fractions are expressed in a standardised format called the "floating point format".

Let us first consider decimal numbers.

For Ex: Given 25.16

the floating point format is 0.2516×10^2

Ex: $0.025 \Rightarrow 0.25 \times 10^{-1}$

Ex: $1024 \Rightarrow 0.1024 \times 10^4$

Mantissa Exponent

Ex: $0.35 \Rightarrow 0.35 \times 10^0$

Ex: Convert 0.35 to binary

$$\begin{array}{l} 0.35 \times 2 \\ \downarrow 0 \leftarrow 0.70 \times 2 \\ 1 \leftarrow 1.40 \times 2 \\ 0 \leftarrow 0.80 \times 2 \\ 1 \leftarrow 1.60 \times 2 \\ 1 \leftarrow 1.20 \end{array}$$

continues endlessly!

$\therefore (0.01011\dots)_2 = (0.35)_{10}$

We will need to round off!

Homework

- convert $(0.01011)_2$ back to decimal and check error!

Let us now consider binary fractions

Ex: Convert 25.625 into binary floating point form.

$$\begin{array}{r} 2 \overline{) 25} \quad \downarrow \text{Reminder} \\ 2 \overline{) 12} \quad 1 \\ 2 \overline{) 6} \quad 0 \\ 2 \overline{) 3} \quad 0 \\ \quad 1 \quad 1 \end{array} \quad (0.625) = (0.101)_2$$

(see pages)

$(25)_{10} = (11001)_2$

$\therefore 25.625 = (11001.101)_2$

Floating point form is

0.11001101×2^{-5} or

Mantissa $\Rightarrow 11001101$ $(10^{-101})_2$

Exponent $\Rightarrow -101$