U3A Maths

19-Mar-2024 Term 1 / Week 8

Binary Fractions

Review

- A negative binary number can be represented as follows:
  (1) Left most bit used as sign.
  $0 \Rightarrow$ +ive $1 \Rightarrow$ −ive
  (2) 1's complement form
  Left most bit indicates the sign! $0 \Rightarrow$ +ive $1 \Rightarrow$ −ive
  (3) 2's complement form
  Left most bit indicates the sign! $0 \Rightarrow$ +ive $1 \Rightarrow$ −ive

- options (1) & (2) result in different bit patterns for +0 & −0 [Considering 4 bit m/c]
  Ex (1) Signed Number 0000 & 1000
  (2) 1's complement 0000 & 1111
- The 2's complement has only +0. Hence, 2's complement form is used as the standard form in modern digital computers. for negative numbers
- As a bonus, the extra bit pattern is used to represent additional negative number.
  For 4 bit m/c $1000 \xrightarrow[\text{compl}]{2's} +1000$ (−8)!

Binary Fractions

- Fractional part in binary is usually expressed using a binary point! (Analogous to decimal point)
- For example, consider

0.101

2^{-1} ($\frac{1}{2}$) 2^{-2} ($\frac{1}{4}$) 2^{-3} ($\frac{1}{8}$)

Note: $2^{-1} = \frac{1}{2} = 0.5$

$2^{-2} = \frac{1}{2^2} = \frac{1}{4} = 0.25$

$2^{-3} = \frac{1}{2^3} = \frac{1}{8} = 0.125$

Hence, the value in decimal form is

$= 1 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3}$

$= 1 \times 0.5 + 0 \times 0.25 + 1 \times 0.125$

$= 0.625$

$\therefore (0.101)_2 = (0.625)_{10}$

- Ex.2 Convert 11.11 to decimal

Convert whole and fractional parts separately.

$$(1\ 1\ .\ 1\ 1)_2 \quad (2^1\ 2^0\ .\ 2^{-1}\ 2^{-2})$$

$= 1\times2^1 + 1\times2^0 . \underbrace{1\times2^{-1}}_{0.5} + \underbrace{1\times2^{-2}}_{0.25}$

$= (3.75)_{10}$

- Just as an exercise, let us convert a decimal fraction into binary fraction

Ex: Convert $(0.625)_{10}$ to binary

0.625×2

1 ← $\not{0}.250$ ↓ starting

0.25×2 from top-

0 ← $\not{0}.5 \times 2$ the value is

1 ← $\not{0}$ $(0.101)_2 = (0.625)_{10}$

Ex: Convert 0.35 to binary

0.35×2

0 ← $\not{0}.70 \times 2$ ↓

1 ← $\not{1}.40 \times 2$

0 ← $\not{0}.80 \times 2$

1 ← $\not{1}.60 \times 2$

1 $\not{1}.20$

↳ continues endlessly!

$\therefore (0.01011\ldots)_2 = (0.35)_{10}$

We will need to round off!

Homework

- convert $(0.01011)_2$ back to decimal and check error!

Floating Point Number

- In computers, numbers with fractions are expressed in a standardised format called the "floating point format"
- Let us first consider decimal numbers.

For Ex: Given 25.16
the floating point format is
0.2516×10^2

Ex: $0.025 \Rightarrow 0.25 \times 10^{-1}$

Ex: $1024 \Rightarrow \underbrace{0.1024}_{\text{Mantissa}} \times 10^{\overbrace{4}^{\text{Exponent}}}$

Ex: $0.35 \Rightarrow 0.35 \times 10^0$

- Let us now consider binary fractions

Ex: Convert 25.625 into binary floating point form.

		← Reminder
2 | 25		
2 | 12	1	
2 | 6	0	
2 | 3	0	
	1	1

$(0.625) = (0.101)_2$ (See page 5)

$(25)_{10} = (11001)_2$

$\therefore 25.625 = (11001.101)_2$

Floating point form is

0.11001101×2^{-5} or $(10^{-101})_2$

$\therefore$ Mantissa $\Rightarrow$ 11001101

Exponent $\Rightarrow -101$