U3A Maths

14-May-2024 Term 2 / Week 3

Complex Numbers
(An Alternative View)

Ref: 'seshveda.com'

- A Complex number is the most general form of a number.
- In other words, the decimal number system used in practice is a particular case of the corresponding complex number form.

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- Human beings invented (used) the number system to corent the physical objects,
  Ex: 5 apples, 6 oranges etc.
- Initially, the number system were essentially positive numbers which was extended to include fractions.
- Let us first consider "a set of positive numbers" including rational & irrational fractions
  (½), (¼), (3/2)... (⅓, 2/3, ...)

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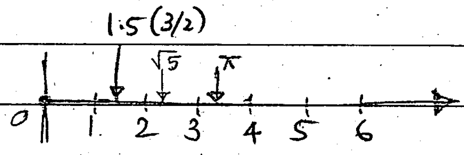
- They can be represented geometrically as below:



- A set of numbers is ~~said~~ to be 'closed' for a given arithmetic operation provided the resulting number is also a part of the set.
- The set of positive numbers is closed for addition, multiplication, division and square root.
  Ex: $3+5=8$, $6\times8=48$, $3/2=1.5$, $\sqrt{5}=2.236$ etc
- A Set of positive numbers is not necessarily closed for subtraction. Ex: $5-3=2$ ✓ $3-5=-2$ ✗

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- Negative numbers are not a part of the above set.
- Hence, we need to extend the set to include negative numbers
  (History) →
- Decimal System / positive numbers
  ⇒ Fibonacci 1202 AD
  "Liber Abaci - Modus Indorum"
  (Book of calculations - Method of Indians)
- Negative numbers
  ⇒ Girolamo Cardano 1545 AD.
  "Ars Magna" (Great Art!)
- Brahma Gupta - 620 AD
  Detailed manuscript including Zero, positive (Fortune), Negative (Debt) numbers and quadratic equations.

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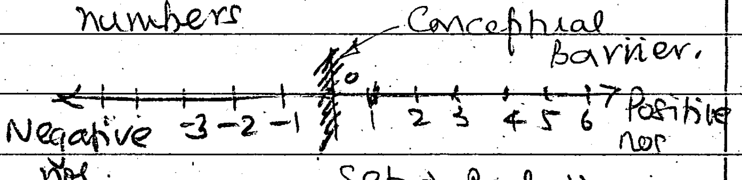
- Acceptance of "negative" numbers took almost 200 to 300 years, since they were considered to be impractical or fictitious numbers!
- One has to cross a mental barrier to accept negative numbers



Set of Real Nos.

- Incidentally, it took 1200 years for the Western world to accept 'zero' and change from Roman numerals to Decimal numbers!

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- Let us called the set which includes negative numbers as a "Set of Real Numbers". (We are stuck with this terminology!)
- A set of Real Numbers is closed for addition, subtraction, multiplication and division.
- But it is not necessarily closed for square root.

Ex: $\sqrt{4} = \pm 2$ ✓ $\sqrt{3} = \pm 1.732$ ✓

$\sqrt{-3} = ??$ $\sqrt{-1} = ??$

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- In fact, we cannot represent square root of negative numbers geometrically on the above figure for Real nos. (Hence, Des Cartes called them "Imaginary" nos.)
- Cardano encountered the square root of negative numbers while solving cubic equations, as given below:

$$x^3 = 15x + 4 \quad (x = ?)$$

- He knew that there must be at least one "Real Root" for cubic equations.
- This can be checked by plotting the function $f(x) = x^3 - 15x - 4$)

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- He had also developed an equation to obtain the "Real Root". Using his equation, he ended up with the following result!

$$x = \sqrt[3]{2 + \sqrt{-121}} + \sqrt[3]{2 - \sqrt{-121}}$$

- His colleague Bombelli solved the problem by setting

$$\sqrt[3]{2 + \sqrt{-121}} = (a + bi)$$
$$\sqrt[3]{2 - \sqrt{-121}} = (a - bi)$$

(He assumed $i = \sqrt{-1}$)

After some manipulation he established $a = 2$ & $b = 1$

$\therefore$ He had the answer!

$$x = (a + \cancel{bi}) + (a - \cancel{bi}) = 2a = \underline{4}\ !!$$