U3A Maths

3-JUN-2024 Term 2/Week 6

Complex Number Arithmetic

- We saw that square root of a complex number is a complex number.

- In other words, we can logically solve the problem of square root of a negative number by expressing it as a complex number in Euler's form.

  For Ex: $-4 = 4e^{\pm i180^\circ}$

  $\therefore \sqrt{-4} = \sqrt{4e^{\pm i180^\circ}} = 2e^{\pm i90^\circ}$

  $= 2(\cos 90^\circ \pm i\sin 90^\circ) = \pm 2i$

Complex Number Forms

- There are 3 forms for expressing a complex number.

Euler's Form

- Eventhough this form is not commonly found in text books (except while introducing Euler's equation), as per this author, it is the most logical form and helps to conceptualise complex numbers.

  For Ex:

  $A = \underset{\text{Magnitude always positive}}{5} \cdot e^{i60^\circ}$ ← Angle w.r.t. the Ref.

- The above form is mathematically complete and it helped us to calculate the square root of a complex number.

Cartesian Form

- Cartesian form is the r.h.s. of the Euler's Equation

$$A \cdot e^{i\theta} = A(\cos\theta + i\sin\theta)$$
$$= A\cos\theta + iA\sin\theta)$$

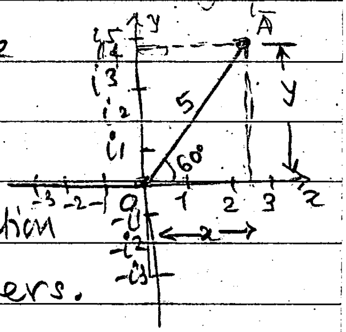
For Ex:

$$\bar{A} = 5e^{i60}$$
$$= 5(\cos 60^\circ + i\sin 60^\circ)$$
$$= (2.5 + i4.33) = (x+iy)$$

Note:
$\bar{A}$ represents Complex Variable
$|A|$ or A represents the Magnitude.

- Note that Cartesian form makes it easier to plot complex numbers on $x-y$ plots.

- We will soon see that cartesian form is useful for addition & subtraction of complex numbers.



Polar Form

- Polar form is a simplified form of Euler's form.

  For Ex:

  $\bar{A} = 5e^{i60} \Rightarrow 5\angle 60^\circ$

- Polar form is very popular due it's simplicity. However, the polar form is mathematically and conceptually incomplete.

- The traditional text books extensively use 'Polar' & 'Cartesian' form with adequately explaining The Euler's Eqn. & Euler's form. This can cause confusion and leads to rote learning!

Complex Multiplication & Division

- Complex number multiplication & division is simple & straight form, when we use Euler's form.

Ex:

$\bar{A} = 4e^{i60^\circ}$ $\quad \bar{B} = 2e^{i30^\circ}$

- Euler's Form

$$\bar{A} \times \bar{B} = 4e^{i60^\circ} \times 2e^{i30^\circ}$$
$$= 8e^{i(60^\circ+30^\circ)}$$
$$= 8e^{i90}$$
$$= (8\cos 90^\circ + i\, 8\sin 90^\circ)$$
$$= (0 + i8)$$

- In polar form

$$\bar{A} \times \bar{B} = 4\angle 60^\circ \times 2\angle 30^\circ$$
$$= 8\angle 60^\circ+30^\circ = 8\angle 90^\circ$$

Note: this has to be Rote Learnt.

Also: $\bar{A} / \bar{B} = 4e^{i60^\circ}/2e^{i30^\circ}$

$$= 4e^{i(60^\circ-30^\circ)} = 4e^{i30}$$

Complex Addition & Subtraction

Ex: $\bar{A} = 4e^{i60}$; $\bar{B} = 2e^{i30^\circ}$

$$\bar{A} + \bar{B} = 4e^{i60^\circ} + 3e^{i30^\circ}$$

- This is similar to $(a^x + b^y)$!

Hence, it is not feasible!!

- We need to use Cartesian Form

$$\bar{A} = 4e^{i60} = 4(\cos 60^\circ + i\sin 60^\circ)$$
$$= (2 + i3.464)$$

Also $\bar{B} = 2e^{i30^\circ} = (1.732 + i1)$

$$\therefore \bar{A} + \bar{B} = (2 + i3.464) + (1.732 + i1)$$
$$= (3.732 + i4.464)$$
$$= \sqrt{(3.732^2 + 4.464^2)} \angle \tan^{-1}\frac{4.464}{3.732}$$
$$= 5.819\angle +50^\circ.1 \text{ or } 5.819\, e^{i50^\circ.1}$$

Gauss's Remark on Complex Number Terminology

- "If this subject has hitherto been considered from the wrong viewpoint and thus enveloped in mystery & darkness, it is largely an unsuitable terminology which should be blamed. Had +1, -1 and $\sqrt{-1}$, instead of being called pos., neg and imaginary unity, been given name say, direct, inverse and lateral unity, there would have any scope for obscurity!"

- Finally do not forget

$$e^{i\pi} = -1$$
or $$e^{i\pi} + 1 = 0 \;!$$