U3A Maths

25-Jun-2024 Term 2 / Week 9

Complex Numbers &

A.C. Circuit Analysis

- Complex numbers help to simplify the solution of Alternating Current (AC) networks.
- Let us first solve Direct Current (DC) networks.
- DC networks have only resistive elements since effect of magnetic and electrostatic fields can be ignored.

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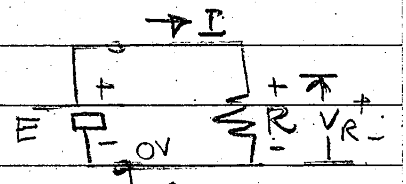
- Ohm's law

States that



Voltage drop (V_R) (=) Current (I) × Resistance (R)

or $V_R = I R$ or $I = \frac{V_R}{R}$.

(For a given resistor the value of R is a constant)

We have

$$E = V_R = I \cdot R$$

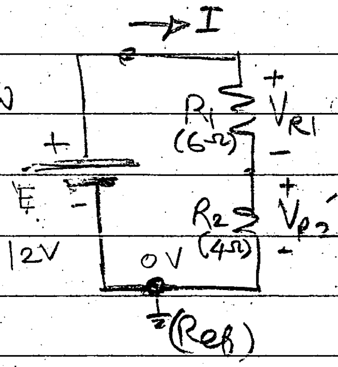
If $V_S = 12\,V$ and $R = 6\,\Omega$

$$I = \frac{E}{R} = \frac{12}{6} = 2A.$$

-3-

- If we have two resistances (R_1 & R_2) are connected in series.

Using Ohm's law we have



$V_{R1} = I R_1$

$V_{R2} = I R_2$

Using Kirchoff's Voltage Law (KVL) – total voltage rise & drop in a closed circuit is zero.

$\therefore +E - V_{R1} - V_{R2} = 0$

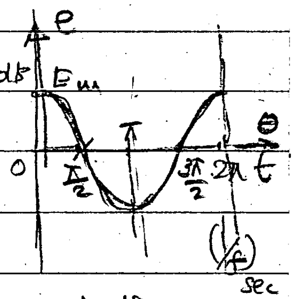
$$E = V_{R1} + V_{R2} = I R_1 + I R_2$$

$$\therefore I = \frac{E}{(R_1 + R_2)} = \frac{12}{6+4} = \underline{\underline{1.2A}}$$

-4-

- An AC source or generator voltage varies sinusoidally w.r.t. time.
- This choice makes A.C. systems more efficient!
- Let us say A.C. voltage is

$$e(t) = E_m \cos(\theta) \text{ Volts}$$



where 'θ' is in radians or degrees.

- But, we need to express this as a function of time.
- Let us say there are "f" cycles in one second.

$\therefore$ 2π radians $\Rightarrow$ $(1/f)$ Seconds

$\therefore$ 1 Second $\Rightarrow$ $2\pi f$ radians

$\therefore$ At any time 't': $\underline{e(t) = E_m \cos(2\pi f \cdot t) \text{ Volts}}$

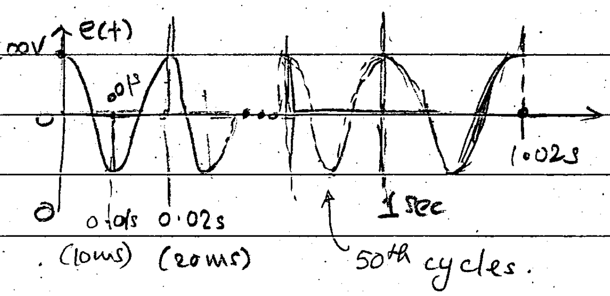
Ex: Given $E_m = 100V$ & $f = 50$ cycles/s. Calculate the voltage at (a) 0.0 sec (b) 10 ms (c) 20 ms (d) 1.02 s

(a) $e(0) = 100 \cdot \cos(2\times\pi\times 50\times 0) = 100V$

(b) $e(10ms) = 100 \cdot \cos(2\pi\times 50\times 10\times 10^{-3}) = -100V$

(c) $e(20ms) = 100 \cdot \cos(2\pi\times 50\times 20\times 10^{-3}) = 100V$

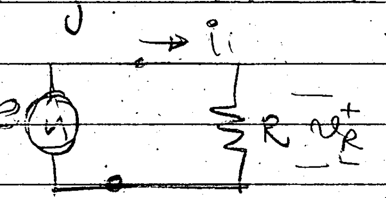
(d) $e(1.02) = 100 \cos(2\pi\times 50\times 1.02) = +100V$



- Hence the above equation can be used to calculate the voltage at any given time!

Resistance

- Solving resistive circuits in AC is straight forward, except that the current also follows the voltage variations.
- Ohm's law is still valid at any given time 't'.



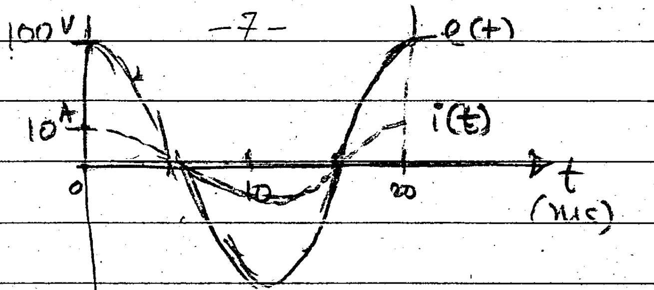
$v_R = iR$

$e = v_R = iR$ or $i = \dfrac{e}{R}$

Note: $e(t) \equiv e$; $v_R(t) \equiv v_R$ & $i(t) = i$. Lower case is used for time varying quantities.

Given $e = E_m \cos(2\pi ft)$ Volts where $E_m = 100V$ $f = 50$ c/s

for $R = 10\Omega$; $i = \dfrac{100 \cos(2\pi ft)}{10} = 10\cos(2\pi ft)$ Amps.



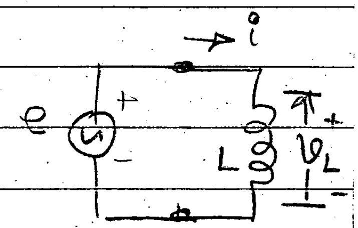
Inductance (Effect Magnetic field)

- A current flow in a conductor produces a corresponding magnetic flux (field). However, in the case of AC, the varying magnetic flux induces a voltage in the same conductor. Such a voltage is given by:

$v_L = L \cdot \dfrac{di}{dt}$ → Faraday's Law

'L' (inductance) is a constant for a given magnetic setup.

- Hence we have:



$e = v_L = L \cdot \dfrac{di}{dt}$

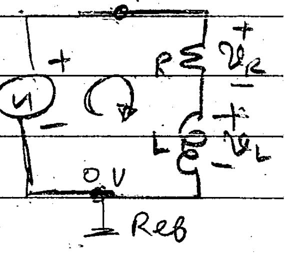
where $e = E_m \cos(2\pi ft)$ volts

$\therefore\ i = \dfrac{1}{L}\int e\,dt = \dfrac{1}{L}\int E_m \cos(2\pi ft)$

$= \dfrac{E_m}{2\pi fL} \sin(2\pi ft)$

A bit difficult but can be solved!

- Now let us consider another circuit. Given $e = E_m \cos(2\pi ft)$, find 'i'



we have:

$v_R = iR$

$v_L = L \cdot \dfrac{di}{dt}$

Also voltage around the loop

$e - v_R - v_L = 0$

$\therefore\ e = v_R + v_L = iR + L \cdot \dfrac{di}{dt}$

How do we solve this for 'i'!