U3A Maths – Fractional Roots

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Calculate $\sqrt[2.5]{2}$

Stewart's Method

$$(2)^{1/2.5}$$

$$Z_K = \sqrt[2.5]{2}\left[\cos\left(\frac{2\pi k}{n}\right) + i\sin\left(\frac{2\pi k}{n}\right)\right]$$

we have : $k = 0, 1, 2, 3, 4, \ldots$ and $n = 2.5$

Polar Method (Add 144°)

$Z_0 = \sqrt[2.5]{2} = 1.3195\angle 0°$

$Z_1 = (-1.0675 + i\,0.7756) = 1.3195\angle 144°$

$Z_2 = (0.4077 - i\,1.2549) = 1.3195\angle 288°$

$Z_3 = (0.4077 + i\,1.2549) = 1.3195\angle 72°$

$Z_4 = (-1.0675 - i\,0.7756) = 1.3195\angle 216°$

$\boxed{Z_5 = (1.3195 + i0) = 1.3195\angle 0°}$

The roots Repeat after this !

Note: Polar angle is $\frac{360°}{2.5} = 144°$

Phil's Method

$$(2)^{1/2.5} = (2)^{2/5} = (4)^{1/5}$$

or $\sqrt[5]{4}$

We have : $k = 0, 1, 2, 3, 4$ and $n = 5$

Polar Method (Add 72°)

$Z_0 = \sqrt[5]{4} = 1.3195\angle 0°$

$Z_1 = (0.4077 + i\,1.2549) = 1.3195\angle 72°$

$Z_2 = (1.0675 + i\,0.7756) = 1.3195\angle 144°$

$Z_3 = (-1.0675 - i\,0.7756) = 1.3195\angle 216°$

$Z_4 = (0.4077 - i\,1.2549) = 1.3195\angle 288°$

$\boxed{Z_5 = (1.3195 + i0) = 1.3195\angle 0°}$

The roots repeat after this !

Note: Polar angle is $\frac{360}{5} = 72°$

Notes:

- Both methods give same results, but not in the same order
- Stewart's method is more straight forward !
- Polar methods makes calculations trivial ! Phil's method is more logical !
- I got carried away with 10/25, because of Stewart's 10/53 for $\sqrt[5.3]{19.7}$
- Phil, I should have simplified 10/25 to 2/5.

Sorry for the confusion caused by me – Sesha