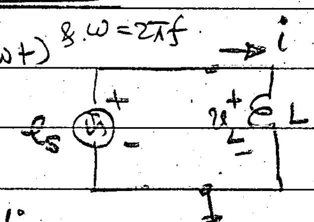
U3A Maths

13-Jul-2024 Term3 / Week4

A.C. Circuit Examples

Inductance (Traditional Method)

Given $e_s = E_{sm}\cos(\omega t)$ & $\omega = 2\pi f$ and L, find 'i'



we have,

$e_s = v_L = L.\frac{di}{dt}$ — Faraday's Law

$\therefore\ i = \frac{1}{L}\int e_s\,dt = \frac{1}{L}\int E_{sm}\cos\omega t\,dt$

$$\boxed{i = \frac{E_{sm}}{\omega L}\sin(\omega t)}$$

Ex.1 Given $E_{sm} = 100V$, $f = 50Hz$ and $L = 10mH$, calculate 'i'.

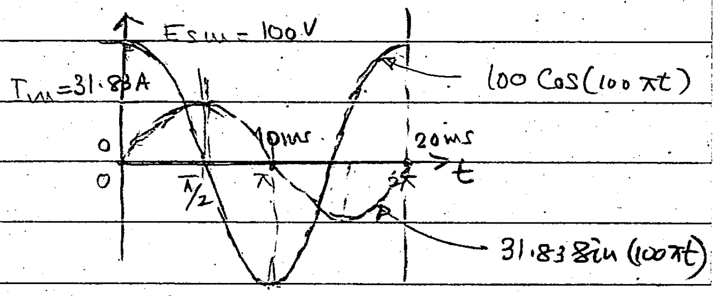
-2-

we have, $E_{sm} = 100V$, $L = 10\,mH = 10\times10^{-3}\,H$

$\omega = 2\pi f = 2\pi\times50 = 100\pi$

$\therefore\ i = \frac{100}{(100\pi)\times(10\times10^{-3})}\sin(\omega t)$

$= 31.83\sin(100\pi t)$ Amps.

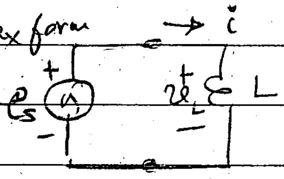


Note that (peak value of) current lags the (peak value of) voltage by $\pi/2$ radians or $90°$.

-3-

Inductance (Complex No. Method)

To express 'e_s' in complex form let



$\bar{E}_s = E_{sm}.e^{j\omega t}$

$= \underbrace{E_{sm}\cos(\omega t)}_{\text{Real Part}} + j\,E_{sm}\sin(\omega t)$

$\therefore\ e_s = \text{Real Part of}\left(\bar{E}_s \text{ or } E_{sm}e^{j\omega t}\right)$

$= E_{sm}\cos(\omega t)$

Using circuit equations

$e_s = v_L = L.\frac{di}{dt}$

$i = \frac{1}{L}\int e_s\,dt$

$= \text{Real Part of}\left[\frac{1}{L}.\int E_{sm}e^{j\omega t}\right]$

$= \text{Real Part of}\left[\frac{1}{j\omega L}.E_{sm}e^{j\omega t}\right]$

-4-

Expressing as complex variables:

$\bar{I} = \frac{\bar{E}_s}{\bar{Z}_L} = \frac{E_{sm}.e^{j\omega t}}{(\omega L)\,e^{j90°}}$

Note: let $\bar{Z}_L = \omega L\,e^{j90°}$

$= \left(\frac{E_{sm}}{\omega L}\right)e^{j(\omega t - 90°)}$

$\therefore\ i = \text{Real Part of }(\bar{I})$

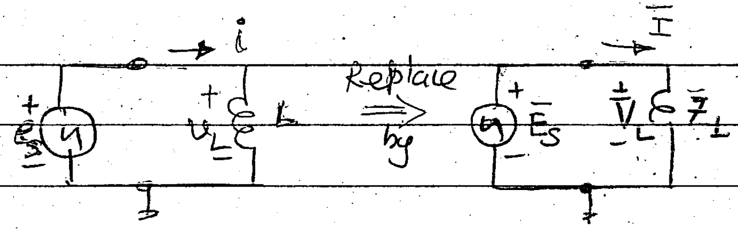
$= \left(\frac{E_{sm}}{\omega L}\right)\cos(\omega t - 90°)$

$$\boxed{i = \left(\frac{E_{sm}}{\omega L}\right)\sin(\omega t)}\ !$$

Ex 2

Solve Example 1 using complex variables;

Let us replace our circuit with complex variables;



In complex form we have

$$\bar{E}_S = 100\, e^{j\omega t} \text{ volts}$$

$$\bar{Z}_L = (\omega L)\, e^{j90^\circ} \text{ ohms}$$
$$= (2\pi f L)\, e^{j90^\circ}$$
$$= (2\pi \times 50 \times 10 \times 10^{-3})\, e^{j90^\circ}$$
$$= (\pi)\, e^{j90} \text{ ohms.}$$

∴ current in complex form

$$\bar{I} = \frac{\bar{E}_S}{\bar{Z}_L} = \frac{100\, e^{j\omega t}}{(\pi)\, e^{j90}}$$
$$= 31.83\, e^{j(\omega t - 90^\circ)} \text{ Amps}$$

∴ the actual current

$$i = \text{Real part of } (\bar{I})$$
$$= \text{Real Part of } [31.83 \cos(\omega t - 90^\circ) + j\, 31.83 \sin(\omega t - 90^\circ)]$$

$$\therefore\ i = 31.83 \cos(\omega t - 90^\circ)$$
$$= \underline{31.83 \sin(\omega t)} \text{ !!}$$

- Same result as before, but no integration was necessary!
- Inductance equation can now be treated as algebraic equation instead of differential equation

- We can replace time dependent equation

$$e_s = v_L = L \cdot \frac{di}{dt}$$

by complex equation

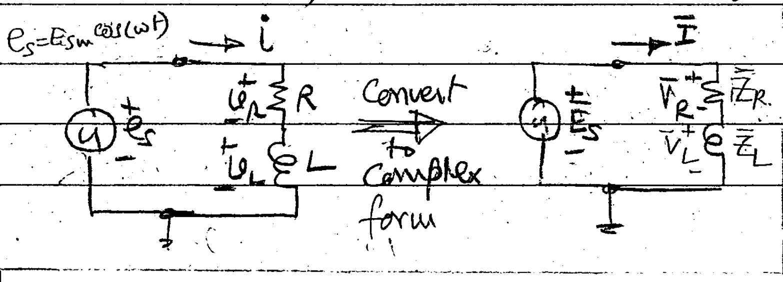
$$\bar{E}_S = \bar{V}_L = \bar{Z}_L \cdot \bar{I}$$

$$\text{or} \quad \bar{I} = \frac{\bar{E}_S}{\bar{Z}_L} = \frac{\bar{V}_L}{\bar{Z}_L}$$

where $\bar{Z}_L = j\omega L$ or $\omega L e^{j90^\circ}$

Ex: 3

Given $E_{sm} = 100V$, $f = 50Hz$, $R = 10\Omega$ & $L = 10mH$, Calculate current 'i'.



We have:
$$\omega = 2\pi f = 2\pi \times 50 = 100\pi$$
$$\bar{E}_S = E_{sm}\, e^{j\omega t} = 100\, e^{j100\pi t} \text{ V}$$
$$\bar{Z}_R = 10\Omega = 10\, e^{j0} \ \Omega$$
$$\bar{Z}_L = \omega L\, e^{j90^\circ} = 3.1416\, e^{j90} \ \Omega$$

Using K.V.L. for the circuit

$$\bar{E}_S = \bar{V}_R + \bar{V}_L = \bar{I}\bar{Z}_R + \bar{I}\bar{Z}_L$$

$$\therefore\ \bar{I} = \frac{\bar{E}_S}{\bar{Z}_R + \bar{Z}_L} = \frac{100\, e^{j\omega t}}{10\, e^{j0} + 3.1416\, e^{j90^\circ}}$$
$$= \frac{100\, e^{j\omega t}}{(10 + j0) + (0 + j3.1416)}$$
$$= \frac{100\, e^{j\omega t}}{10.482\, e^{j17.44^\circ}} = \underline{9.54\, e^{j(\omega t - 17.44^\circ)}} \text{ Amps.}$$

∴ The actual current is

$$i = \text{Real Part of } [\bar{I}]$$
$$= \underline{9.54} \cos(\omega t - 17.44^\circ) \text{ Amps}$$

∴ current (peak) lags voltage (peak) by 17.44°!