

3-Sep-2024

Term 3/Week 7

Logarithms

- Addition/subtraction is much simpler than Multiplication/Division

$2+3=5$ $2 \times 3=6$ ✓

$25+42=67$ $25 \times 42 = 1050$ ✓

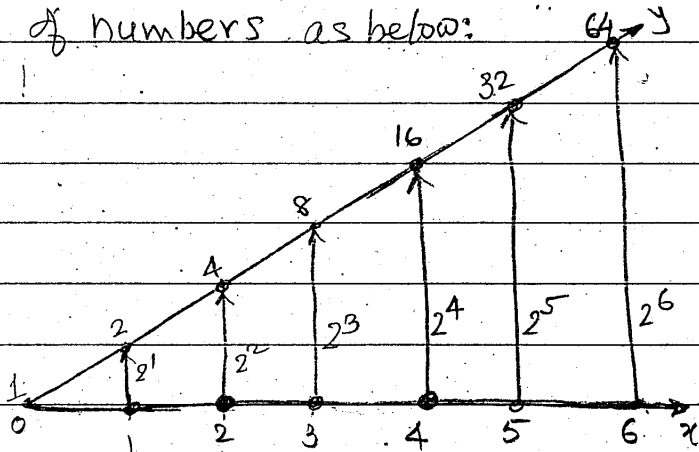
$251+425=676$ $251 \times 425 = ?$

Also $425/251 = ?$

- The question is, can we somehow "transform" multiplication/Division to Addition/subtraction?

- 3 -

- Let us consider 2 sets of numbers as below:



- It can be noted that 'y' values correspond to the 2^{th} power of 2

i.e., $y = 2^x$
easily

- We can extend the above for increasing values of 'x'.

- 4 -

- Let us try multiplication of values along 'y' axis

Ex: $4 \times 16 = 64$

- Alternatively, pick the corresponding values on the 'x' axis

$\frac{x}{4} \Rightarrow x$

$16 \Rightarrow 4$

Add the 'x' values, i.e., $2+4 = \underline{\underline{6}}$

Now, pick the value of 'y' for the corresponding to "sum"

i.e., $\frac{x(\text{sum})}{6} \quad y(\text{result})$
 $\underline{\underline{64}} \leftarrow \text{Answer!}$

• The question is what happens for in between values!

• We can extend the correspondence as shown in the "table" below:

<u>x</u>	<u>y = 2^x</u>
0	2 ⁰ = 1
0.5	2 ^{0.5} = √2 = 1.414
1	2 ¹ = 2
1.5	2 ^{1.5} = 2√2 = 2.828
2	2 ² = 4
2.5	2 ^{2.5} = 4√2 = 5.656
3	2 ³ = 8
3.5	2 ^{3.5} = 8√2 = 11.314 (!)
4	2 ⁴ = 16

• Let us try another Ex.

y ⇒ 2.828 × 5.656 = 15.995

Corresponding 'x' values are

<u>y</u>	<u>x</u>
2.828 ⇒	1.5
5.656 ⇒	2.5
∴ Sum	= <u>4.0</u>

Hence the product is

x = 4 ⇒ y = 16 ✓

• Obviously the accuracy depends number of digits used for the table.

<u>x</u>	<u>y</u>
4.5	2 ^{4.5} = 16√2 = 22.628
5	2 ⁵ = 32
5.5	2 ^{5.5} = 32√2 = 45.255
6	2 ⁶ = 64
	<u>etc</u>

• With some difficulty we can extend the table for 2^{0.25}, 2^{0.125}, etc.

• Remember that John Napier took about 20 years to develop his table with 7 digit precision!

• When once the table is ready, then multiplication/division becomes easier!

Another Ex:

22.628 - 11.314 = 2

We have

<u>y</u>	<u>x</u>
22.628 ⇒	4.5
11.314 ⇒	3.5

Difference (for division) = 1.0

x = 1.0 ⇒ y = 2 ✓

• Home work:

(1) 5.656 × 11.314

(2) 64 ÷ 22.628

• Hence, we essentially have a "logarithmic" table with a base of '2'.

Logos ⇒ Ratio
Antithmos ⇒ Number