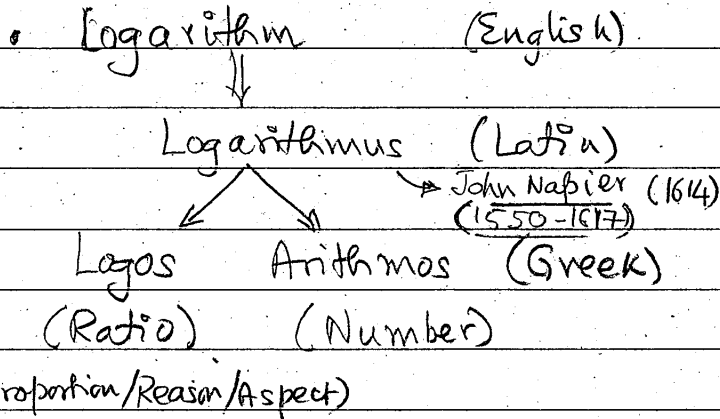


10-Sep-2024

Term 3/Week 8

Logarithms - Bases



• The main objective of the logarithm is to "transform" multiplication & division as addition & subtraction respectively

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x:	0	1	2	3	4	5	6	7	8
y = 2 ^x :	1	2	4	8	16	32	64	128	256

• Ex: $y \Rightarrow 8 \times 16 = 128$

$x = \log_2(y) \Rightarrow 3 + 4 = 7 \Rightarrow 128$

(x) \Rightarrow y (From Table)

• The table works well, but the problem in establishing logarithm of "in-between" values of 'y' is difficult.

• Note that ^{the} calculation 2^x is quite straightforward, but such a table is not sufficient for general arithmetic calculations

• Let

$$y = a^x$$

We define, the logarithmic function symbolically as below:

$$x = \log_a(y)$$

ie, 'x' is the logarithm of 'y' for the given base 'a'.

• Last week we prepared a partial logarithmic table using a base value of '2' In other words,

$$y = 2^x \quad \& \quad x = \log_2(y)$$

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• Our choice of the base value of '2' was arbitrary. In fact there is nothing "holy" about the base value of '2'.

• We can choose the base value as per our convenience.

• Let us say we choose a base of "1,000".

The choice of this base was done by Jost Burgi (Swiss, clock maker, 1552-1632)

• In fact, Jost Burgi used a base of '2' to illustrate the concept, namely, "arithmetic" & "geometric" progression using powers of 2.

Let us prepare the table

x	$y = (1.0001)^x$
0	1.0
1	1.00010000
2	1.00020001
3	1.00030003
4	1.00040006
5	1.00050010
6	1.00060015
7	1.00070021
...	...
10	1.00100045
...	...
15	1.00150105
...	...
25	1.00250300
...	...
2250	1.025230863
...	...
100,000	22015.4561
100,001	22017.6576

Compare this table with Burgi's Table!

Lastly $999,999 \times 2.67444 \times 10^{43}$
 $1,000,000 \times 2.67471 \times 10^{43}$

Note that it quite simple to obtain table using the formula

$$y_{n+1} = y_n \times 1.0001$$

Example 1

$$y \Rightarrow 1.00100045 \times 1.00150105 = 1.00250300$$

$$x \Rightarrow 10 + 15 = 25$$

$\downarrow y$
1.00250300

Ex.2 $1.00150105 / 1.00100045 = 1.00050010$

Burgi Table (Notes)

- Need to add decimal point for 'x' values, ie, $10 \Rightarrow 1.0$, $500 \Rightarrow 750.0$
- 'y' values are multiplied by 10^8

Burgi's table (reconstruction)

$x \rightarrow$	0	50.0	100.0	150.0
\downarrow	0	500	1000	1500
0	0	100000000	100501227	101004966
1	10	100000000	100501227	101004966
2	20	100000000	100501227	101004966
3	30	100000000	100501227	101004966
4	40	100000000	100501227	101004966
5	50	100000000	100501227	101004966
6	60	100000000	100501227	101004966
7	70	100000000	100501227	101004966
8	80	100000000	100501227	101004966
9	90	100000000	100501227	101004966
10	100	100000000	100501227	101004966
11	110	100000000	100501227	101004966
12	120	100000000	100501227	101004966
13	130	100000000	100501227	101004966
14	140	100000000	100501227	101004966
15	150	100000000	100501227	101004966
16	160	100000000	100501227	101004966
17	170	100000000	100501227	101004966
18	180	100000000	100501227	101004966
19	190	100000000	100501227	101004966
20	200	100000000	100501227	101004966
21	210	100000000	100501227	101004966
22	220	100000000	100501227	101004966
23	230	100000000	100501227	101004966
24	240	100000000	100501227	101004966
25	250	100000000	100501227	101004966
26	260	100000000	100501227	101004966

Burgi Table Example Multiplication (See the circled values)

Multiply following 'y' values:
 $y \Rightarrow 100551488 \times 101055479$

\downarrow

'x' (From Table) $55 + 105 = 160$

Result: 101612787 From Table Find 'y'

Burgi Table gives only the first 9 digits of the product
 For ex: $1.0001 \times 10^8 \times 1.0001 \times 10^8 = 1.00020001 \times 10^8$
 It is left to the user to adjust the 'decimal point' position.

Say we have: $10055.1488 \times 10105.5479$
 $= 101612787. \dots$
 First 9 significant digits of the product