# U3A Maths

10-Sep-2024 Term3/Week8

## Logarithms - Bases

- Logarithm (English)

  ⇓

  Logarithmus (Latin) → John Napier (1614) (1550-1617)

  ↙ ↘

  Logos (Ratio) (Proportion/Reason/Aspect) Arithmos (Number) (Greek)

- The main objective of the logarithm is to "transform" multiplication & division as addition & subtraction respectively

-2-

- Let

$$y = a^x$$

We define, the logarithmic function symbolically as below:

$$x = \log_a(y)$$

- ie, 'x' is the logarithm of 'y' for the given base 'a'.

- Last week we prepared a partial logarithmic table using a base value of '2'. In otherwords,

$$y = 2^x \quad \& \quad x = \log_2(y)$$

-3-

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $x$: | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| $y=2^x$: | 1 | 2 | 4 | (8) | (16) | 32 | 64 | (128) | 256 |

(3 + 4 → 7)

- Ex: $y \Rightarrow 8 \times 16 = 128$

  $x = \log_2(y) \quad 3 + 4 = 7 \Rightarrow (128)$

  $(x) \Rightarrow y$ (From Table)

- The table works well, but the problem is establishing logarithm of "in-between" values of 'y' is difficult.

- Note that the calculation $2^x$ is quite straightforward, but such a table is not sufficient for general arithmetic calculations

-4-

- Our choice of the base value of '2' was arbitrary. In fact there is nothing "holy" about the base value of '2'.

- We can choose the base value as per our convenience.

- Let us say we choose a base of "1.0001".
  The choice of this base was done by Jost Burgi (Swiss, Clock Maker, 1552-1632)

- In fact, Jost Burgi used a base of '2' to illustrate the concept, namely, "arithmetic" & "geometric" progression using powers of 2.

- Let us prepare the table

| $x$ | $y = (1.0001)^x$ |
|---|---|
| 0 | 1.0 |
| 1 | 1.00010000 |
| 2 | 1.00020001 |
| 3 | 1.00030003 |
| 4 | 1.00040006 |
| 5 | 1.00050010 |
| 6 | 1.00060015 |
| 7 | 1.00070021 |
| ... | ... |
| → 10 | 1.00100045 |
| ... | ... |
| → 15 | 1.00150105 |
| ... | ... |
| → 25 | 1.00250300 |
| ... | ... |
| 2,250 | 1.25230863 |
| ... | ... |
| 100,000 | 22015.4561 |
| 100,001 | 22017.6576 |

(Compare this table with Burgi's Table!)



| | $x$ | $y$ |
|---|---|---|
| Lastly | 999,999 | $2.67444 \times 10^{43}$ |
| | 1,000,000 | $2.67471 \times 10^{43}$ |

- Note that it quite simple to obtain table using the formula

$$y_{n+1} = y_n \times 1.0001 \; !$$

- Example 1

$y \Rightarrow 1.00100045 \times 1.00150105 = 1.00250300$

$x \Rightarrow 10 + 15 = 25$

↓ $y$ 1.00250300

Ex. 2 $1.00150105 / 1.00100045 = 1.00050010$

- Burgi Table (Notes)
  - Need to add decimal point for '$x$' values, ie, $10 \Rightarrow 1.0$, $500 \Rightarrow 50.0$ etc
  - '$y$' values are multiplied by $10^8$.



Buergi's table (reconstruction)

$x$ → (0, 50.0, 100.0, 150.0); $x$ ↓ (0–26)

| | | 0 | 500 | 1000 | 1500 |
|---|---|---|---|---|---|
| 0 | 0 | 100000000 | 100501227 | 101004966 | 101511230 |
| 1 | 10 | ....10000 | ....11277 | ....15067 | ....21381 |
| 2 | 20 | ....20001 | ....21328 | ....25168 | ....31534 |
| 3 | 30 | ....30003 | ....31380 | ....35271 | ....41687 |
| 4 | 40 | ....40006 | ....41433 | ....45374 | ....51841 |
| 5 | 50 | ....50010 | ....51488 | ....55479 | ....61996 |
| 6 | 60 | ....60015 | ....61543 | ....65584 | ....72152 |
| 7 | 70 | ....70021 | ....71599 | ....75691 | ....82310 |
| 8 | 80 | ....80028 | ....81656 | ....85798 | ....92468 |
| 9 | 90 | ....90036 | ....91714 | ....95907 | 101602627 |
| 10 | 100 | 100100045 | 100601773 | 101106017 | ....12787 |
| 11 | 110 | ....10055 | ....11834 | ....16127 | ....22949 |
| 12 | 120 | ....20066 | ....21895 | ....26239 | ....33111 |
| 13 | 130 | ....30078 | ....31957 | ....36351 | ....43274 |
| 14 | 140 | ....40091 | ....42020 | ....46465 | ....53438 |
| 15 | 150 | ....50105 | ....52084 | ....56580 | ....63604 |
| 16 | 160 | ....60120 | ....62150 | ....66695 | ....73770 |
| 17 | 170 | ....70136 | ....72216 | ....76812 | ....83938 |
| 18 | 180 | ....80153 | ....82283 | ....86930 | ....94106 |
| 19 | 190 | ....90171 | ....92351 | ....97048 | 101704275 |
| 20 | 200 | 100200190 | 100702420 | 101207168 | ....14446 |
| 21 | 210 | ....10210 | ....12491 | ....17289 | ....24617 |
| 22 | 220 | ....20231 | ....22562 | ....27411 | ....34790 |
| 23 | 230 | ....30253 | ....32634 | ....37533 | ....44963 |
| 24 | 240 | ....40276 | ....42707 | ....47657 | ....55138 |
| 25 | 250 | ....50300 | ....52782 | ....57782 | ....65313 |
| 26 | 260 | ....60325 | ....62857 | ....67908 | ....75490 |



- Burgi Table Example Multiplication (See the circled values)
- Multiply following 'y' values:

$y \Rightarrow 100551488 \times 101055479$

'$x$' (From Table) $55 + 105 = 160$

Result: 101612787 (From Table Find "y")

- Burgi Table gives only the first 9 digits of the product

For Ex: $1.0001 \times 10^8 \cdot \times 1.0001 \times 10^8 = 100020001 \times 10^8$

- It is left to the user to adjust the 'decimal point' position.

Say we have: $10055.1488 \times 10105.5479$

$= 101612787.xxxxxxx$

First 9 significant digits of the Product