U3A Maths {Week 3 - No class}

5-Nov-2024 Term 4 / Week 4

Logarithms Theory

- We saw (in Week 1) that Napier coined the term "Logarithm" and provided a table of values and their logarithms. (1614 AD) (≈10 million values & 20 years!)
- He also provided an abbreviated table relating logarithmic values with "Sine/Cos" values from 90° to 0° in steps 1′ (Minute) (Total Size: 90×60 = 540 Values!)

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- The "Sine/Cos" table was an act of brilliance! It helped to relate the method of "Prosthaephaeresis" which was used for carrying out multiplication & division to logarithms.

$$\sin(A) \cdot \sin(B) = \frac{1}{2}\left(\cos(A-B) + \cos(A+B)\right)$$

- Hence, it helped mathematicians to transit from "prosthaephaeresis" to "Logarithms". Logarithm was rediculously simpler! - Any dummy could see it!!

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- Henry Briggs (1561 - 1631) [Educated in Cambridge & Professor at Gresham College, London & then at Oxford!] met Napier and they agreed on the need to reformulate logarithmic table
- Briggs published his first table of decimal logarithms (base of 10) in 1617. His logarithmic table is the one which is popularly used today!

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- Brigg's table had 14 digit precision, compared with the popular present day log table which has 5 digits!
- Brigg's methods of calculation is too complicated, so we will not go there!
- As per Brigg's method

$$\log_{10} 1 = 0 \Rightarrow 10^0 = 1$$

$$\log_{10} 10 = 1 \Rightarrow 10^1 = 10$$

Recall,

Given $y = 10^x$ then $x = \log_a (y)$

the base $a = 10$ for Briggs table.

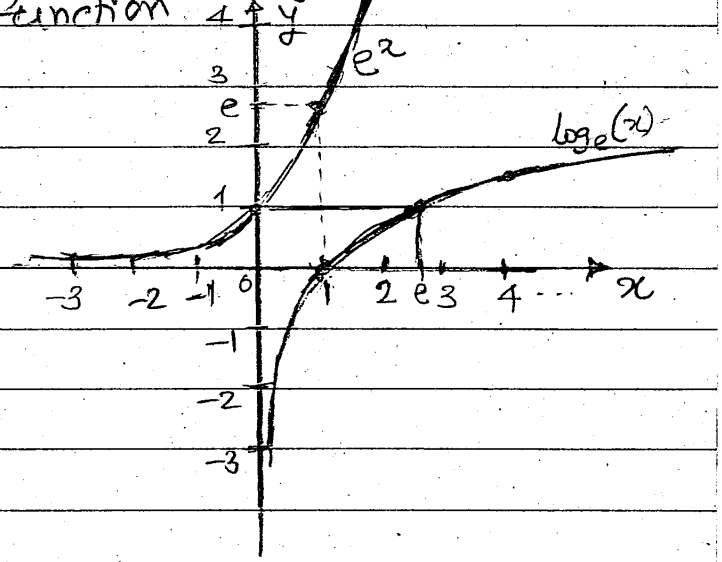
- It is interesting that neither Napier or Briggs defined the logarithmic function in a formal way as above. They essentially provided a (logarithmic) table to enable mult./division!

- It was Leonhard Euler (1707 – 1783), who actually defined the 'log' function. However, he used the base as the exponential number (e) [Note: $e \simeq 2.718$]

As per Euler,

If $y = e^x$ then $x = \log_e(y)$

The "log" function is complementary to "Exponential" function



- Even though $\log_e(y)$ is popular in mathematics, we will stick to base 10 for further discussions.

Laws of logarithms!

1st Law – Product Rule

$$\log_a(m \times n) = \log_a(m) + \log_a(n)$$

Proof ?

let $x = \log_a(m)$ $\therefore$ $m = a^x$

let $y = \log_a(n)$ $\therefore$ $n = a^y$

$\therefore$ $(m \times n) = a^x \cdot a^y$

$(m \times n) = a^{(x+y)}$

Taking logarithms, we have

$(x+y) = \log_a(m \times n)$

$\therefore$ $\log_a(m \times n) = \log_a(m) + \log_a(n)$

2nd law – Quotient Rule

$$\log_a(m/n) = \log_a(m) - \log_a(n)$$

Proof $\Rightarrow$ Home work!

3rd Law – Power Rule

$$\log_a(m^n) = n \cdot \log_a(m)$$

Proof (?)