U3A Maths

19-Nov-2024 Term4 / week6

Logarithms - Examples

Let us first review, logarithm Power Rule;

$$\log_a(m^n) = n \cdot \log_a(m)$$

Proof?

A simple proof is as below:

$$\log_a(m^n) = \log_a(\underbrace{m \times m \times \cdots \times m}_{\text{'n' terms}})$$

$$= \underbrace{\log_a(m) + \log_a(m) + \cdots + \log_a(m)}_{\text{'n' terms}}$$

$$\therefore \log_a(m^n) = n \cdot \log(m)$$

Ex.1 Find the $\log_{10}(0.052)$

We have $0.052 = 5.2 \times 10^{-2}$

$$\therefore \log_{10}(0.052) = \log_{10}(5.2) + \log_{10}(10^{-2})$$

$$= 0.7160 + (-2)\log_{10}(10)$$

$$= 0.7160 + (-2)$$

Note: This corresspands to 10^{-2}, So it is preferable to keep it as it is!

We write

$$\underbrace{\bar{2}}_{\text{Exponent}}.\underbrace{7160}_{\text{Mantissa}}$$

Ex.2 Find the antilog ($\bar{2}.7160$)

$$\bar{2} + 0.7160$$

antilog ⇓ ⇓ antilog from table

$$10^{-2} \times 5.200$$

$$= 5.2 \times 10^{-2} = 0.052!$$

Ex3 Calculate 25.6×0.052

We have: $25.6 = 2.56 \times 10^{1}$

$0.052 = 5.20 \times 10^{-2}$

$$= \log_{10}((10^{1} \times 2.56) \times (10^{-2} \times 5.2))$$

$$= \log_{10}(10^{1} \times 2.56) + \log_{10}(10^{-2} \times 5.2)$$

$$= 1.4082 + \bar{2}.7160$$

Adding:

$$\begin{array}{r} 1.4082 \\ \bar{2}.7160 \\ \hline 0.1242 \end{array}$$

Taking Antilog = 1.331

Using Calculator ⇒ 1.3312!

Note: After some experience, we should be able to write the log values (Exponent & Mantissa) by inspection!

Ex. 4 Calculate $25.6 / 0.052$

$25.6 \Rightarrow 2.56 \times 10^1$

$0.052 \Rightarrow 5.2 \times 10^{-2}$

$\log_{10}\left((10^1 \times 2.56) \div (10^{-2} \times 5.2)\right)$

$= 1.4082 - \bar{2}.7160$

we have $\overset{-1}{1}.4082$

$- \bar{2}.7160$

$(?) .6922$

Mantissa $-1 + 1 - (-2) = +2$

$\therefore$ we have 2.6922

Taking Antilog $\Rightarrow 4.922 \times 10^2$

$= 492.2$

From calculator $\Rightarrow 492.3$

Ex. 5 Calculate $\sqrt{2}$

$\sqrt{2} = (2)^{1/2}$

Using log: $\log_{10}(2^{1/2})$

$= \frac{1}{2} \log_{10}(2)$

$= \frac{1}{2} \times 0.3010 = 0.1505$

Taking antilog $= 1.415$

(From calculator $1.4142!$)

Of course, the roots are ± 1.415

Ex. 6 Calculate $\sqrt[3]{27}$

We have $(27)^{1/3}$

Taking log: $\log_{10}(27^{1/3})$

$= \frac{1}{3} \log_{10}(27) = \frac{1}{3} \log(2.7 \times 10^1)$

$= \frac{1}{3} \times 1.4314 = 0.4738$

(Note: we can use log again for division)
(Homework!)

Taking antilog, we have

$0.4738 \xrightarrow[\text{antilog}]{} 2.977!$

$\simeq 3.0$

- We have 3 roots for $\sqrt[3]{27}$
- We still need De Moivre's theorem to calculate the roots!

Using De Moivre's theorem

Roots are $\Rightarrow 3 . e^{i\frac{2\pi k}{n}}$

where $k = 0$ to $(n-1)$

ie, $= 0, 1, 2$

We have 3rd root, $\therefore n = 3$

$R_1 = 3 . e^{i\frac{2\pi . 0}{3}} = 3 \angle 0°$

$R_2 = 3 . e^{i\frac{2\pi \times 1}{3}} = 3 \angle 120°$

$R_3 = 3 . e^{i\frac{2\pi \times 2}{3}} = 3 \angle 240°$

Home Work

Calculate $\sqrt[2.5]{4}$ using logarithms and also all the roots using De Moivre's theorem.