(No Class)
(week 7 & 8)

U3A Maths

10-Dec-2024 Term 4 / Week 9

Logarithms & Roots

Ex. 1 Calculate $\sqrt[3]{27}$

$$\sqrt[3]{27} = (27)^{1/3}$$

Using logarithms

$$\tfrac{1}{3} \log_{10} (27)$$

$$= \tfrac{1}{3} (1.4314)$$

$$= 0.477133\ldots$$

Taking antilog, we get Root = 3

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- We know that $\sqrt[3]{27}$ has 3 roots.
- The above result provides only one root, which is the real root!
- How about the other 2 roots?
- We need De Moivre's theorem to calculate the other two roots.
- In general, the roots can be complex numbers and hence De Moivre's theorem is defined for a complex number

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For a given complex number,

Say, $A.e^{i\theta}$

The n^{th} root is given by

$(Ae^{i\theta})^{1/n}$ where can be an integer (2, 3, 4, ...) or a fraction (1.5, 2.6, ...)

As per De Moivre's theorem, the roots are

$$R_K = (A)^{1/n} e^{i\left(\frac{2\pi K+\theta}{n}\right)}$$

Where K = 0, 1, 2, ... (n-1)

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For our example, ie $(27)^{1/3}$

Expressing in complex form

$$\left(27.e^{i0^\circ}\right)^{1/3}$$

In other words, we have,

$A = 27$, $\theta = 0^\circ$ and $n = 3$

and $K = 0, 1$ & 2

$\therefore$ the roots are

$$R_0 = (27)^{1/3}.\ e^{i\left(\frac{2\pi\times 0+0^\circ}{3}\right)}$$

$$= (27)^{1/3}.\ e^{0} = (27)^{1/3}$$

$$= 3 \text{ or } 3e^{i0^\circ}$$

What we have calculated using logarithms is $R_0 = 3$!!

$$R_1 = (27)^{1/3} \cdot e^{i\left(\frac{2\pi \cdot 1 + 0}{3}\right)}$$

$$= 3 \cdot e^{i 2\pi/3} = 3 e^{i\, 360/3}$$

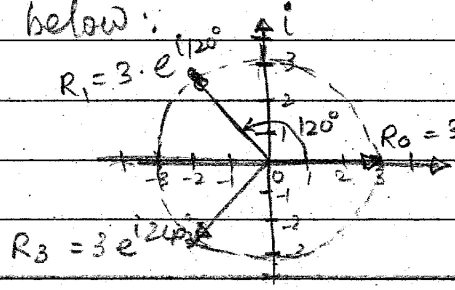
$$= 3 e^{i 120^\circ}$$

$$R_2 = (27)^{1/3} \cdot e^{i\left(\frac{2\pi \cdot 2 + 0^\circ}{3}\right)}$$

$$= 3 \cdot e^{i 4\pi/3} = 3 \cdot e^{i 720^\circ/3}$$

$$= 3 \cdot e^{i 240^\circ}$$

The roots can be plotted as below:



We can check the results.

$$(R_0)^3 = (3)^3 = 27$$

$$(R_1)^3 = \left(3 e^{i120^\circ}\right)^3 = 27 \cdot e^{i360^\circ}$$
$$= 27$$

$$(R_2)^3 = \left(3 e^{i240^\circ}\right)^3 = 27 \cdot e^{i720^\circ}$$
$$= 27$$

Note: $e^{i360^\circ} = \cos(360^\circ) + i\sin(360^\circ)$
$= 1 + i0 = 1$

& $e^{i720^\circ} = \cos(720^\circ) + i\sin(720^\circ)$
$= 1 + i0 = 1$

Ex.2 Calculate $\sqrt{4} = (4)^{1/2}$

using logarithms

$$\tfrac{1}{2} \log_{10}(4) = \tfrac{1}{2} \times 0.6021$$
$$= 0.30105$$

Using Antilog of 0.3010
Root = 2

In complex form, we have

$$\left(4 \cdot e^{i0^\circ}\right)^{1/2}$$

∴ using De Moivre's theorem

$A = 4$, $\theta = 0^\circ$, $n = 2$

& $K = 0, 1$

$$R_0 = (4)^{1/2} \cdot e^{i\left(\frac{2\pi \cdot 0 + 0^\circ}{2}\right)}$$

$$= 2\, e^{i0} = 2$$

$$R_1 = (4)^{1/2}\, e^{i\left(\frac{2\pi \cdot 1 + 0^\circ}{2}\right)}$$

$$= 2 \cdot e^{i\pi} = 2 \cdot e^{i180^\circ}$$

$$= 2\left(\cos(180^\circ) + i\sin(180^\circ)\right)$$
$$= 2(-1 + i \times 0)$$
$$= -2$$

∴ The roots are

$R_0 = +2 = 2e^{i0}$

& $R_1 = -2 = 2e^{i180^\circ}$