

11-Mar-2025

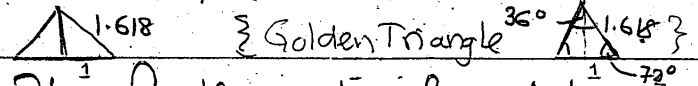
Term 1 / Week 6

Fibonacci Numbers

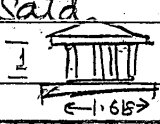
- Fibonacci conceived the Fibonacci series of numbers by a hypothetical problem involving reproduction of rabbits  
 $\Rightarrow 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, \dots$
- Surprisingly, such a series along with the ratio of sequential numbers (Golden Ratio:  $\phi \approx 1.618$ ) was found to occur frequently in 'nature' and "its evolution".

• Eventhough, it is called Fibonacci numbers, the Golden Ratio has been in use in earlier civilizations. For example, they are found in the Pyramids, Greek and other ancient structures.

• The slant height of the Pyramid to half the base length is extremely close to the Golden Ratio.



• The Parthenon's facade's width and height are said to follow Golden Ratio. (Golden Rectangle)



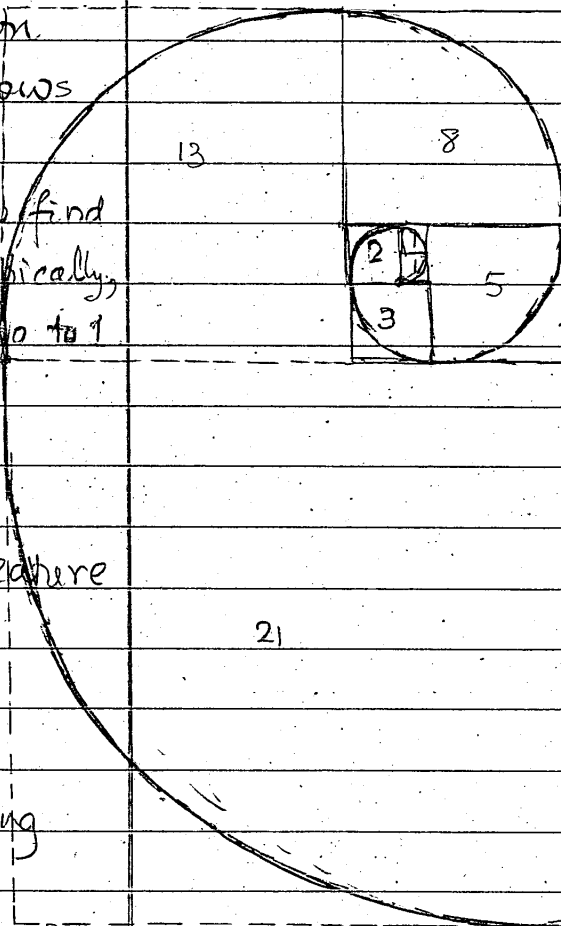
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- The number of leaves on a clover generally follows Fibonacci numbers. For example, it is rare to find a four leaf clover - typically, probability of 5000 to 10,000 to 1

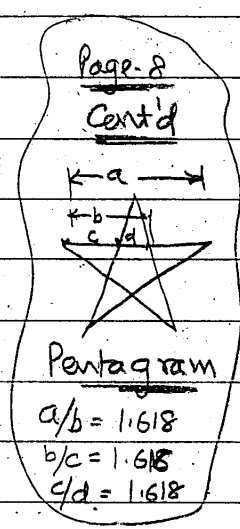
Fibonacci Spiral

- Even more interesting feature of Fibonacci numbers is the Fibonacci spiral.
- It can be constructed using squares based on Fibonacci Numbers.

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• The spiral has been plotted to scale



- The number of Fibonacci spirals of sunflower seeds are 34 (clockwise) and 55 (anticlockwise). This pattern optimises the space for the seeds to grow.

- Similarly, the spirals on pine cones and pineapples are 8 and 13 spirals!

- Incidentally, the spiral formation in the galaxies also correspond to Fibonacci spirals!

- The above "quadratic equation" can be used to find the exact value of  $\phi$

$$\phi = 1 + \frac{1}{\phi} \Rightarrow \phi^2 = \phi + 1$$

Hence,  $\phi^2 - \phi - 1 = 0$

Using quadratic Eqn. Solution.

$$\phi = \frac{-b \pm \sqrt{b^2 - 4ac}}{2} \quad \text{wh. we } a=1, b=-1 \& c=-1$$

$$\phi = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2 \times 1}$$

$$= \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}$$

### Golden Ratio ( $\phi$ )

$13/8 = 1.625$      $34/21 = 1.619...$   
 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89...  
 $8/5 = 1.6...$      $21/13 = 1.615...$      $89/55 = 1.618...$

- Golden Ratio has some interesting features

$$\phi = 1.618...$$

$$1/\phi = 0.618$$

$$(\phi - 1) = 0.618$$

Also, we have

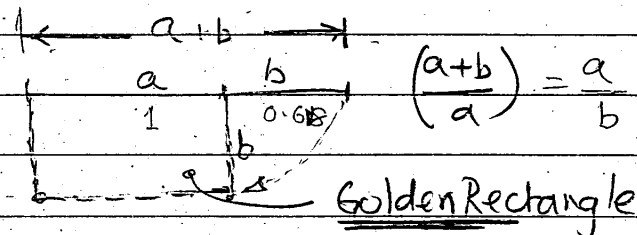
$$\phi = 1 + 1/\phi$$

$\therefore$  the roots are

$$\phi_1 = \frac{1 + \sqrt{5}}{2} \quad \text{and} \quad \phi_2 = \frac{1 - \sqrt{5}}{2}$$

$$\approx 1.618... \quad \quad \quad \approx -0.618...$$

- Golden ratio is obtained by dividing a line, so that the (long part  $\div$  short part) =  $\phi = 1.618$



- Golden Angle — is the angle separating leaves & florets, so that they get maximum sunlight & space

$$\frac{a}{b} = \frac{(a+b)}{b} \quad \theta = 137.5^\circ$$